

Optimal Debt Maturity and Firm Investment*

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Abstract

This paper introduces a maturity choice to the standard model of firm financing and investment. Long-term debt renders the optimal firm policy time-inconsistent. Lack of commitment gives rise to debt dilution. This problem becomes more severe during downturns. We show that cyclical debt dilution generates the observed counter-cyclical behavior of default, bond spreads, leverage, and debt maturity. It also generates the pro-cyclical term structure of corporate bond spreads. Debt dilution renders the equilibrium outcome constrained-inefficient: credit spreads are too high and investment is too low. In two policy experiments we find the following: (1) an outright ban of long-term debt improves welfare in our model economy, and (2.) debt dilution accounts for 84% of the credit spread and 25% of the welfare gap with respect to the first best allocation.

Keywords: firm financing, investment, debt maturity, credit spreads, debt dilution.

JEL classifications: E22, E32, E44, G32.

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1. Introduction

Borrowing costs for U.S. firms increased dramatically during the Great Recession. The default rate on corporate bonds increased from 0.13% in 2007 to 2.48% in 2009.¹ What drives these fluctuations in credit spreads and default rates? The standard approach in the literature is to address this question using a model of one-period debt. Empirically, firms rely heavily on long-term debt.² This paper studies the link between credit market frictions and economic activity using a model in which firms are allowed to issue both short-term and long-term liabilities.

In our model, firms choose leverage by trading off the tax advantage of debt against the potential cost of default. Firms also choose the maturity of their debt. Short-term debt has the disadvantage that its entire amount needs to be rolled-over each period. This is costly because of a transaction cost on the bond market.

Our model replicates the stylized facts of U.S. firm financing.³ Default rates, bond spreads, leverage, and debt maturity all increase during downturns. At the same time, the difference between long-term and short-term bond spreads (the term structure) falls. This paper shows that a single economic mechanism can account for these empirical facts: Cyclical debt dilution.

Debt dilution arises because long-term debt renders the optimal debt issuance policy time-inconsistent.⁴ The firm disregards part of the total potential costs of default, because it does not internalize the effect of its actions on the value of previously issued bonds. Through this mechanism, debt dilution induces the firm to lever up. Higher leverage increases the risk of default at any point of the business cycle.

A novel result of our paper is that the intensity of debt dilution varies over the business cycle. During recessions, investment and the amount of newly issued debt are low, but the ratio of previously issued debt to newly issued debt is high. Because the firm disregards a large fraction of the total potential costs of default, it chooses to run a particularly high risk of default during downturns. As we show below, this renders the default rate, credit spreads, leverage, and debt maturity counter-cyclical.

While the equilibrium in a model of one-period debt is constrained-efficient, this is no longer true once we allow firms to issue both short-term and long-term liabilities.

¹Gilchrist and Zakrajsek (2012) measure an increase in the average credit spread over U.S. treasuries from less than 1.5 percentage points in 2007 to almost 8 percentage points in 2009 for a sample of 5,982 senior unsecured bonds issued by non-financial firms. Adrian, Colla, and Shin (2012) document for a different sample that the average cost of newly issued bonds increased from about 1.5 percentage points in 2007 to more than 4 percentage points in 2009. Data on default rates is from Giesecke, Longstaff, Schaefer, and Strebulaev (2014).

²For U.S. non-financial corporate firms 1984-2016, the average share of long-term liabilities (with term to maturity above on year) is 67%. Gilchrist and Zakrajsek (2012) measure for the years 1973-2011 an average term to maturity of long-term bonds of 11.3 years. Adrian et al. (2012) find that the average maturity of all newly issued bonds fluctuates 1998-2011 between roughly 5 and 15 years.

³We present the cyclical patterns of U.S. corporate firm financing in Section 3.

⁴Debt dilution also plays a key role in quantitative models of sovereign debt. See Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), or Hatchondo, Martinez, and Sosa-Padilla (2016). Bonds issued by U.S. firms commonly include debt covenants which restrict firm policy. In Section 5.5.4, we provide an overview of empirically observed types of debt covenants.

Debt dilution renders the equilibrium outcome constrained-inefficient. Future borrowing exerts an externality on the buyers of newly issued long-term debt. Rational investors pay lower bond prices because firms cannot commit to abstain from debt dilution in the future. Since spreads are inefficiently high, investment is inefficiently low.

The contribution of our paper can be broken down into three parts. In the first part of the paper, we use a simple two-period model to derive analytical results on the cyclical role of debt dilution in driving credit spreads and default rates. We use these analytical results to interpret the numerical findings of our fully dynamic model economy. In our view, these analytical results are also helpful to interpret other numerical findings from the emerging literature on long-term debt (e.g. Gomes, Jermann, and Schmid (2016)).

Models of long-term debt often are solved using an ‘inner loop/outer loop’ procedure. This involves computing the complete bond price schedule for all possible states and actions. This price schedule is then used to compute the optimal policy (e.g. Hatchondo and Martinez (2009)). As a second contribution of our paper, we re-formulate the firm problem in a way which expresses equilibrium bond prices as a function of choice variables and future firm policy. Equilibrium bond prices and today’s firm policy are computed in a single step. This reduces the number of necessary computations and allows for a faster and more precise solution.

The third contribution of our paper is quantitative. Within a fully dynamic economy, we show that cyclical debt dilution generates the observed counter-cyclical behavior of default, bond spreads, leverage, and debt maturity. It also generates the pro-cyclical term structure of corporate bond spreads. We consider two policy experiments which both eliminate debt dilution: (1.) a ban of long-term debt, and (2.) a debt covenant which helps firms to internalize the cost of debt dilution. Both policies improve welfare by reducing credit spreads and increasing investment. We find that debt dilution accounts for 84% of the credit spread and 25% of the welfare gap with respect to the first best allocation. The welfare gain from eliminating debt dilution corresponds to a decrease in the corporate tax rate of 2.5 percentage points.

In Section 2 we briefly survey some related literature. Section 3 documents the stylized facts of cyclical U.S. corporate firm financing. In Section 4 we describe a two-period economy and derive analytical results on the cyclical role of debt dilution. We use these results to interpret our findings from a fully dynamic model economy presented in Section 5. We describe an efficient solution method for this model and study its behavior. We focus on firms’ response to aggregate shocks and the welfare cost of debt dilution. Concluding remarks follow. Formal proofs are deferred to the appendix.

2. Related Literature

To the best of our knowledge, firms’ optimal maturity choice over the business cycle has not been formally studied before. We aim at closing this gap in the literature and find that debt dilution plays a key role for this trade-off.

While the choice between short-term debt and long-term debt is usually absent from models of firm financing and investment, several papers consider an exogenous maturity

structure with long-term debt. In Gomes and Schmid (2016), firms are required to buy back previously issued long-term bonds before issuing new debt. Caggese and Perez (2015) assume that debt is fixed over the life-span of the firm. Setups like these rule out debt dilution by assumption.

Also Miao and Wang (2010) and Gourio and Michaux (2012) study a firm problem with long-term debt. The authors present numerical results on the counter-cyclicality of credit spreads and default without an explicit discussion of debt dilution. We contribute analytical and numerical results which identify the cyclical role of debt dilution. Identifying this role is important because it allows to assess the potential benefits of policy measures which help firms overcome their time-inconsistency problem. A second important difference to our model is that these papers do not consider short-term debt. This assumption is restrictive since a maturity choice allows firms to respond to and mitigate distortions which arise from long-term debt.

In a model with nominal debt, Gomes et al. (2016) show that shocks to inflation change the real burden of long-term debt and thereby distort investment decisions. As in Miao and Wang (2010) and Gourio and Michaux (2012), the role of debt dilution is not explicitly discussed and there is no maturity choice. At the end of section 4.4.3, we briefly discuss how our analytical results relate to the numerical findings from Gomes et al. (2016).

As an exception with respect to the rest of the literature, Crouzet (2016) studies a model in which firms have access both to short-term and long-term debt. In contrast to our paper, firms cannot raise funds by selling equity. Furthermore, the author studies a stationary distribution of firms, whereas we study firms' maturity choice over the business cycle.

We find that debt dilution generates counter-cyclical credit spreads and default rates. Many models without long-term debt do not share this feature. Gomes, Yaron, and Zhang (2003) show that the financial accelerator model without equity issuance (e.g. Carlstrom and Fuerst (1997)) generates a pro-cyclical default rate.⁵ Covas and Den Haan (2012) find a pro-cyclical default rate in a model with costly equity issuance.

While the role of debt dilution has not been explored by the literature on firm financing and investment, it has enjoyed a lot of attention from the literature on sovereign debt. Sachs and Cohen (1982) illustrate debt dilution in a three-period model. A recent wave of quantitative studies finds that debt dilution is important to explain the magnitude and the volatility of credit spreads on sovereign debt. Hatchondo and Martinez (2009) numerically show that debt dilution amplifies the cyclicity of spreads. In Chatterjee and Eyigungor (2012), a sovereign borrower uses long-term debt in spite of debt dilution to reduce the risk of rollover crises. Using a similar setup, Hatchondo and Martinez (2013) find that the optimal maturity choice itself is time-inconsistent. This is also true in our model. Aguiar, Amador, Hopenhayn, and Werning (2016) study a model of sovereign debt in which the competitive equilibrium allocation is Pareto-efficient once one disregards the owners of outstanding long-term debt. Chatterjee and Eyigungor (2015) propose a computationally tractable seniority arrangement. Hatchondo et al.

⁵See also the discussion of the costly state verification model in Quadrini (2011).

(2016) quantify the welfare benefits from different debt covenants.

Arellano and Ramanarayanan (2012) report that the maturity of sovereign debt shortens if spreads increase. Their model generates this pattern because they assume that the exogenous default punishment falls during downturns. This shifts the bond price schedules and reduces the endogenous debt limit in a way which favors short-term debt. In our model, the maturity of firm debt rises if spreads increase. This is because our model does not feature a default punishment which is exogenously tied to the business cycle. Spreads and debt maturity only move over the business cycle because of debt dilution. The ratio of old debt to new debt increases during downturns. Debt dilution becomes stronger which causes both spreads and debt maturity to increase in our model.

The models on sovereign debt cited above are endowment economies. Debt dilution affects credit spreads but these spreads do not affect output. In our model, spreads affect firm investment and output. For this reason, the impact of debt dilution on welfare is very different. Furthermore, models of sovereign debt generate counter-cyclical spreads through an exogenous pro-cyclical default punishment. In contrast, we study a production economy and analytically demonstrate that debt dilution alone can generate counter-cyclical credit spreads. Endogenous investment is key for this mechanism.

3. Empirical Facts

In this section, we document the business cycle behavior of several financial variables for non-financial corporate firms in the US. As recommended by Jermann and Quadrini (2012), we include observations starting in the first quarter of 1984.⁶

Default Rate. Figure 1 plots the yearly default rate on corporate bonds against real GDP 1984-2012. Variables are de-trended using the Hodrick-Prescott filter. The default rate spikes during all three recessions of the sample period. The counter-cyclical behavior of default is confirmed by the business cycle statistics reported in Table 1.

	Corr(x, GDP)	$\frac{\sigma_x}{\sigma_{\text{GDP}}}$	ρ_x
GDP	1.00	1.00	0.40
Corporate Value Added	0.93	1.81	0.44
Default Rate	-0.51	0.49	0.02

Table 1: Business Cycle Statistics (yearly data)

Note: Sample Period is 1984-2012. The default rate is from Giesecke et al. (2014). It is the ratio of ‘total par value of outstanding non-financial corporate bonds which are in default’ over ‘total par value of outstanding non-financial corporate bonds’. Nominal GDP and Gross Value Added of non-financial corporate firms are from the Flow of Funds. They are deflated using the CPI and they are in logs. Cyclical components are obtained using the Hodrick-Prescott filter with a smoothing parameter of 6.25 as recommended by Ravn and Uhlig (2002) for yearly data. The second column shows the correlation with GDP, the third column shows the relative standard deviation with respect to GDP, the last column shows the 1st order autocorrelation coefficient.

⁶Jermann and Quadrini (2012) motivate this choice with the beginning of the Great Moderation and with regulatory changes in U.S. financial markets at that time.

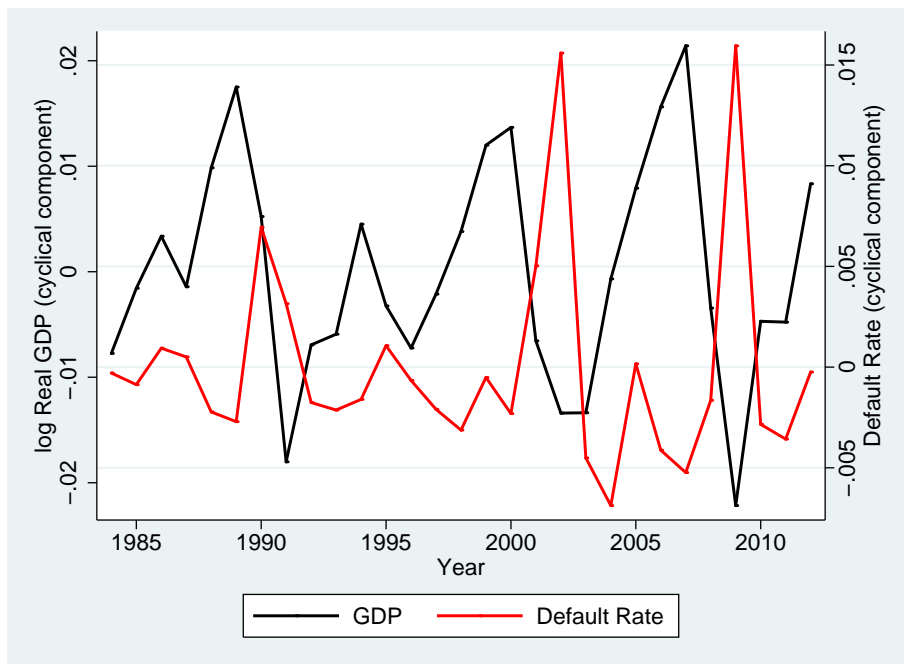


Figure 1: Corporate Default Rate over the Business Cycle.

Note: The default rate is from Giesecke et al. (2014). It is the ratio of 'total par value of outstanding non-financial corporate bonds which are in default' over 'total par value of outstanding non-financial corporate bonds'. Nominal GDP is from the Flow of Funds. Real GDP is calculated using the Consumer Price Index from the Bureau of Labor Statistics. It is in logs. Cyclical components are obtained using the Hodrick-Prescott filter with a smoothing parameter of 6.25 as recommended by Ravn and Uhlig (2002) for yearly data.

Leverage. For all remaining variables data is available on a quarterly frequency. The upper panel of Figure 2 plots corporate leverage over the business cycle. Leverage is defined as total liabilities as a fraction of total assets. Especially during the time period 2000-2011, a strong negative co-movement between output and leverage is apparent. During recent downturns, corporate firms have reduced their assets faster than their liabilities. This has caused leverage to rise in recessions. In Table 2 we calculate business cycle statistics using yearly data 1984-2015.⁷ The correlation between output and leverage is -0.75 .

Long-term Liabilities Share. In the lower panel of Figure 2 the maturity structure of corporate liabilities is plotted against the business cycle. It shows long-term liabilities (with remaining maturity of more than one year) as a fraction of total corporate liabilities. During downturns, the share of long-term liabilities increases sharply as firms reduce short-term liabilities faster than their long-term liabilities. Table 2 shows a correlation between output and the share of long-term liabilities of -0.69 .

Corporate Bond Spreads. Data on corporate bond spreads is only available starting from the first quarter of 1997. The upper panel of Figure 3 shows how yields on corporate

⁷In our numerical exercise in Section 5, the model period is a year. In order to compare our quantitative findings with the data, we calculate business cycle statistics using yearly data.

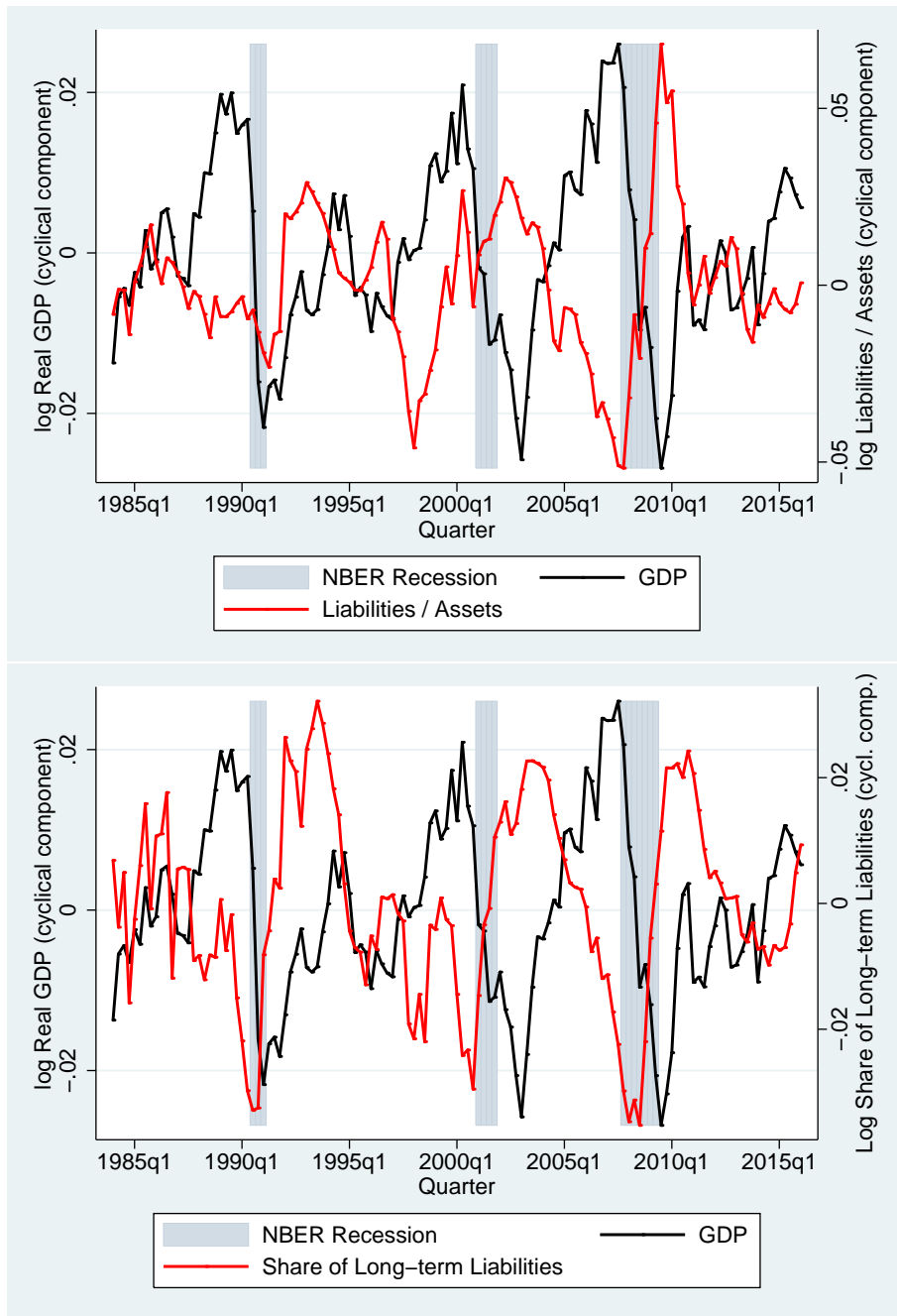


Figure 2: Leverage and the Share of Long-term Liabilities.

Note: Total Assets, Short-term Liabilities, and Total Liabilities of non-financial corporate firms are from the Flow of Funds. Variables are in logs. Cyclical components are obtained using the Hodrick-Prescott filter with a smoothing parameter of 1600. Grey bars indicate NBER recessions.

	Corr(x ,GDP)	$\frac{\sigma_x}{\sigma_{\text{GDP}}}$	ρ_x
GDP	1.00	1.00	0.42
Corporate Value Added	0.95	1.77	0.47
Corporate Investment	0.76	4.39	0.44
Liabilities / Assets	-0.75	1.95	0.30
Long-term Liabilities Share	-0.69	1.47	0.33

Table 2: Business Cycle Statistics (yearly data)

Note: Sample Period is 1984-2015. All variables are from the Flow of Funds. They deflated using the CPI and they are in logs. Cyclical components are obtained using the Hodrick-Prescott filter with a smoothing parameter of 6.25. The second column shows the correlation with GDP, the third column shows the relative standard deviation with respect to GDP, the last column shows the 1st order autocorrelation coefficient.

bonds perform relative to treasury bills of identical maturity. A clear counter-cyclical behavior is detectable for bonds of all maturities. Table 3 shows a negative correlation between output and bond spreads of different maturities.

Term Structure. The lower panel of Figure 3 shows the difference between spreads on long-term bonds (remaining maturity of 5 – 7 years, in green, and more than 15 years, in blue) and spreads on bonds of remaining maturity between 1 and 3 years. This difference is called the term structure of bond spreads. The term structure declines during downturns as short-term spreads increase faster than long-term spreads. Table 3 shows a positive correlation between output and the term structure of corporate bond spreads.

	Corr(x ,GDP)	$\frac{\sigma_x}{\sigma_{\text{GDP}}}$	ρ_x
GDP	1.00	1.00	0.46
Corporate Value Added	0.95	1.86	0.46
Spread 1-3 years	-0.53	0.67	0.21
Spread 5-7 years	-0.56	0.49	0.24
Spread 15+ years	-0.43	0.35	0.22
Difference 15+ vs 1-3 years	0.57	0.36	0.18
Difference 5-7 vs 1-3 years	0.39	0.20	0.20

Table 3: Business Cycle Statistics (yearly data)

Note: Data on Corporate Bond Spreads is from the St. Louis Fed Data Base (FRED). This is the Bank-of-America Merrill-Lynch US Corporate Option-Adjusted Spread for different remaining terms to maturity: 1-3 years, 5-7 years, and more than 15 years. This index is calculated using bonds of investment grade or higher. GDP and Corporate Value Added are seasonally adjusted, deflated using the CPI, in logs, and de-trended. Cyclical components are obtained using the Hodrick-Prescott filter with a smoothing parameter of 6.25. The second column shows the correlation with GDP, the third column shows the relative standard deviation with respect to GDP, the last column shows the 1st order autocorrelation coefficient.

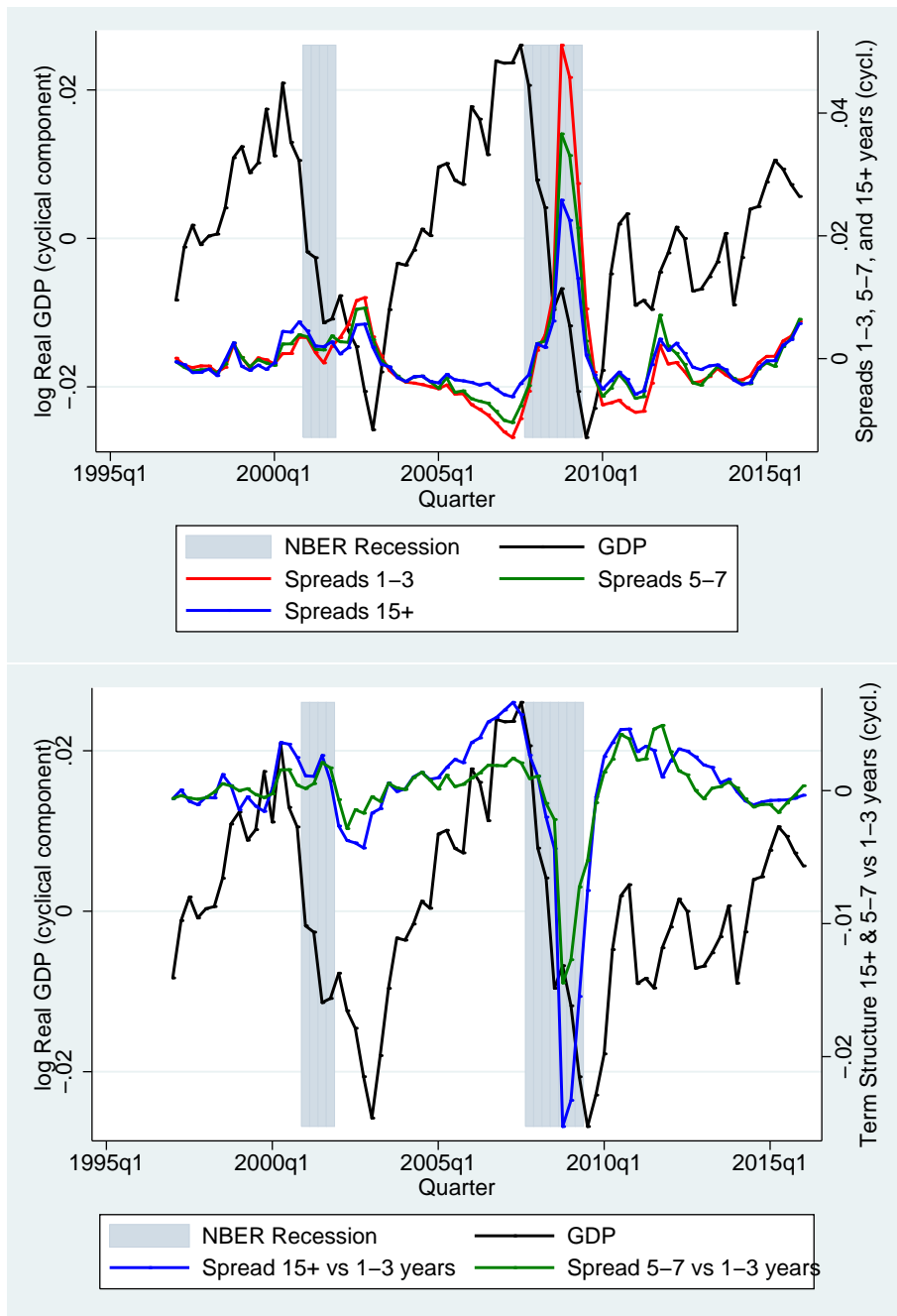


Figure 3: Corporate Bond Spreads and the Term Structure over the Business Cycle.

Note: Data on Corporate Bond Spreads is from the St. Louis Fed Data Base (FRED). This is the Bank-of-America Merrill-Lynch US Corporate Option-Adjusted Spread for different remaining terms to maturity: 1-3 years, 5-7 years, and more than 15 years. This index is calculated using bonds of investment grade or higher. GDP is seasonally adjusted, deflated using the CPI, and in logs. Cyclical components are obtained using the Hodrick-Prescott filter with a smoothing parameter of 1600. Grey bars indicate NBER recessions.

4. Two-period Model

The previous section has established the counter-cyclical behavior of default, bond spreads, leverage, and debt maturity, and the pro-cyclical behavior of the term structure of corporate bond spreads. In this section, we will use a simple two-period setup to derive analytical results on the cyclical properties of default, bond spreads, and leverage. These cyclical properties will crucially depend on whether the firm has outstanding long-term debt.

We solve the problem of a firm which finances its capital stock using equity and debt. The optimal capital structure solves a trade-off between the tax advantage of debt and expected costs of default.

4.1. Setup

There are two periods: $t = 0, 1$. Consider a firm owned by risk-neutral shareholders. In period 1, the firm can use capital k to produce output y using a technology with diminishing returns:

$$y = z f(k). \quad (1)$$

The production function $f(k)$ is increasing and concave. We assume that the marginal product is a constant fraction of its average product: $f'(k) \propto \frac{f(k)}{k}$. To give an example, this property is satisfied by $f(k) = k^\alpha$. Revenue productivity z is known in period 0 when capital k is chosen.

Capital depreciates at rate δ . Earnings before interest and taxes are given as:

$$z f(k) - \delta k + \varepsilon k, \quad (2)$$

where ε is a firm-specific earnings shock. Its value is uncertain at time 0 when capital k is chosen. It follows a probability distribution $\varphi(\varepsilon)$ with mean zero and standard deviation σ_ε . Variations in ε capture unforeseen events which directly affect firm earnings.

Markets are exogenously incomplete. There are two ways to finance the capital stock k . The firm can raise funds using equity and debt.

Definition: Debt. A debt security is a promise to pay one unit of the numéraire good together with a fixed coupon payment c at the end of period 1. The quantity of these bonds is \tilde{b} . Coupon payments $c\tilde{b}$ are fully tax-deductible.

Firm earnings are taxed at rate τ . This implies for shareholders' net worth after production in period 1:

$$q = k - \tilde{b} + (1 - \tau)[z f(k) - \delta k + \varepsilon k - c\tilde{b}]. \quad (3)$$

When the firm chooses \tilde{b} , there may already be a quantity b of bonds outstanding. This is long-term debt which has been issued at some earlier point in time and which matures at date 1 just like the one-period debt which the firm can sell in period 0. The firm can

finance capital in period 0 either using equity or by increasing its debt level \tilde{b} above the initial stock of long-term debt b through the sale of additional bonds:

$$k = e + p(\tilde{b} - b), \quad (4)$$

where e is the quantity of equity invested by shareholders in period 0, and p is the market price of a bond sold by the firm.

Debt is risky as the firm can decide to default on its liabilities after ε is realized in period 1.

Definition: Limited Liability. Whenever the firm's asset value after production is lower than its debt liabilities, full repayment would result in negative net worth q . Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. A fixed fraction ξ of the firm's assets is lost in this case.

Timing

t=0 Given revenue productivity z and an existing stock of debt b , the firm chooses capital k . Capital is financed using equity e and by the revenue $p(\tilde{b} - b)$ from the sale of additional bonds.

t=1 ε is realized. This determines net worth q . The firm decides whether to default.

4.2. Firm Problem

The firm maximizes shareholder value. Limited liability protects shareholders from large negative realizations of the idiosyncratic earnings shock ε . Given a firm's stock of capital k and debt \tilde{b} , there is a unique threshold realization $\bar{\varepsilon}$ which sets shareholders' net worth equal to zero:

$$\bar{\varepsilon}: \quad 0 = k - \tilde{b} + (1 - \tau)[zf(k) - \delta k + \bar{\varepsilon}k - c\tilde{b}]. \quad (5)$$

If ε is smaller than $\bar{\varepsilon}$, shareholders optimally choose to stop paying their liabilities and default.

In period 0, the firm decides on its scale of production k , and its preferred capital structure consisting of equity e and debt \tilde{b} , in order to maximize expected shareholder value at the end of period 1:

$$\max_{k, e, \tilde{b}, \bar{\varepsilon}} \quad -e + \frac{1}{1+r} \int_{\bar{\varepsilon}}^{\infty} [k - \tilde{b} + (1 - \tau)[zf(k) - \delta k + \varepsilon k - c\tilde{b}] \varphi(\varepsilon) d\varepsilon \quad (6)$$

$$\text{subject to:} \quad 0 = k - \tilde{b} + (1 - \tau)[zf(k) - \delta k + \bar{\varepsilon}k - c\tilde{b}]$$

$$k = e + p(\tilde{b} - b),$$

where $\frac{1}{1+r}$ is the shareholders' discount factor. The bond price p depends on the firm's behavior.

4.3. Creditors' Problem

Creditors are risk-neutral. They discount the future at the same rate $\frac{1}{1+r}$ as shareholders. If shareholders stop paying their liabilities and default, the firm's asset value is:

$$\tilde{q}(k, \bar{\varepsilon}) \equiv k + (1 - \tau)[zf(k) - \delta k + \bar{\varepsilon}k]. \quad (7)$$

In case of default, creditors liquidate the firm's assets and receive $(1 - \xi)\tilde{q}(k, \bar{\varepsilon})$.⁸ In period 0, the price which they pay for a bond depends on the firm's characteristics. Creditors break even on expectation:

$$p(\bar{\varepsilon}, \tilde{b}, k) = \frac{1}{1+r} \left[[1 - \Phi(\bar{\varepsilon})](1+c) + \Phi(\bar{\varepsilon}) \frac{(1-\xi)\tilde{q}(k, \bar{\varepsilon})}{\tilde{b}} \right]. \quad (8)$$

Definition: Credit Spreads. For a riskless bond which promises a safe payment of $1+c$ in period 1, creditors pay a price $p_r = \frac{1+c}{1+r}$. In case we have $c=r$, the price of the riskless bond is 1. The price p of a bond issued by the firm is lower than p_r if and only if the probability of default is positive. In this case, there will be a positive credit spread: $1+r_b - (1+r) = \frac{1+c}{p} - (1+r)$, where r_b is the interest rate which firms pay.

4.4. Equilibrium

We solve for the partial equilibrium allocation given the exogenous and fixed discount rate $\frac{1}{1+r}$. In equilibrium, the firm maximizes shareholder value subject to creditors' break-even condition.⁹

4.4.1. Consolidated Problem

It is useful to express the stock of debt \tilde{b} in terms of other variables. From the definition of $\bar{\varepsilon}$ it follows:

$$\tilde{b} = \frac{k + (1 - \tau)[zf(k) - \delta k + \bar{\varepsilon}k]}{1 + (1 - \tau)c}. \quad (9)$$

⁸Even a defaulting firm still services $\bar{\varepsilon}k$. The idea is that ε is realized slowly as the firm monitors its cash flow. Assume that the firm learns about ε by observing a number x with initial value $+\infty$ and which subsequently falls towards ε . In this case, the firm optimally services all payments associated to ε until ε falls below $\bar{\varepsilon}$.

⁹Sometimes more than one bond price satisfies creditors' zero-profit condition. In this case, different bond prices imply different default probabilities. By allowing the firm to directly select the default probability through $\bar{\varepsilon}$, we implicitly assume that the firm sells its bonds to creditors by making a take-it-or-leave-it offer specifying both a price and a quantity of bonds. In this setup, the firm is able to always select the preferred bond price and the preferred default probability. For the phenomenon of multiple bond prices, see Calvo (1988) with an early analysis, Crouzet (2016), or Nicolini, Teles, Ayres, and Navarro (2015) with clarifying results on the role of timing and strategy sets.

Shareholders promise a part of the firm's post-production assets to creditors. This promise $\tilde{b}[1 + (1 - \tau)c]$ consists of two parts: the safe part of firm assets after production, $k + (1 - \tau)[zf(k) - \delta k]$, and a fixed amount of the risky part of earnings, $(1 - \tau)\bar{\varepsilon}k$. From $k = e + p(\tilde{b} - b)$, we derive:

$$e = k - p(\tilde{b} - b). \quad (10)$$

The more funds the firm raises on the bond market, the less equity shareholders have to inject. Using these two expressions, the firm's objective can be re-written as:

$$-k + p(\tilde{b} - b) + \frac{1 - \tau}{1 + r} k \int_{\bar{\varepsilon}}^{\infty} [\varepsilon - \bar{\varepsilon}] \varphi(\varepsilon) d\varepsilon. \quad (11)$$

The firm maximizes the total payoff generated from the investment of capital k in the firm. Since creditors break even on expectation, shareholders appropriate the entire surplus created by the investment both of equity e and debt $p(\tilde{b} - b)$. Dividends $k[\varepsilon - \bar{\varepsilon}]$ are taxed at rate τ . They are positive if ε is higher than the threshold value $\bar{\varepsilon}$. The bond price p is a function of k , $\bar{\varepsilon}$, and \tilde{b} only. This allows us to describe the firm's problem as a maximization problem in two variables:

$$\begin{aligned} \max_{k, \bar{\varepsilon}} \quad & -k + \frac{1}{1 + r} \left[[1 - \Phi(\bar{\varepsilon})](1 + c) + \Phi(\bar{\varepsilon}) \frac{(1 - \xi)\tilde{q}(k, \bar{\varepsilon})}{\tilde{b}} \right] (\tilde{b} - b) \\ & + \frac{1 - \tau}{1 + r} k \int_{\bar{\varepsilon}}^{\infty} [\varepsilon - \bar{\varepsilon}] \varphi(\varepsilon) d\varepsilon \quad (12) \\ \text{subject to: } \quad & \tilde{b} = \frac{k + (1 - \tau)[zf(k) - \delta k + \bar{\varepsilon}k]}{1 + (1 - \tau)c}. \end{aligned}$$

The variable k controls the scale of production. The variable $\bar{\varepsilon}$ decides how much of firm earnings (before interest and taxes) are paid out as taxable dividends and how much is paid to creditors in the form of coupon payments that are fully tax-deductible. The downside of raising $\bar{\varepsilon}$ is that costly default becomes more likely.

4.4.2. First Order Conditions

Consider the special case that $\xi = 1$. A first order condition for an optimal choice of k is:

$$k : \quad -1 + \frac{1 - \Phi(\bar{\varepsilon})}{1 + r} (1 + c) \frac{\partial \tilde{b}}{\partial k} + \frac{1 - \tau}{1 + r} \int_{\bar{\varepsilon}}^{\infty} [\varepsilon - \bar{\varepsilon}] \varphi(\varepsilon) d\varepsilon = 0. \quad (13)$$

A marginal increase in k has opportunity cost 1. A part of the increase in k is financed by the sale of additional bonds. If default is avoided, payments to bond holders increase by $(1 + c) \frac{\partial \tilde{b}}{\partial k}$. This benefit is weighted with the probability that default is avoided $1 - \Phi(\bar{\varepsilon})$. The last term shows that the expected value of dividends grows as the scale of production is increased. For a given value of $\bar{\varepsilon}$, the firm's choice of debt \tilde{b} grows in k according to:

$$\frac{\partial \tilde{b}}{\partial k} = \frac{1 + (1 - \tau)[zf'(k) - \delta + \bar{\varepsilon}]}{1 + (1 - \tau)c}. \quad (14)$$

As the scale of production is increased, the value of debt grows less and less because of diminishing returns in the safe component of firm earnings. The firm's objective is strictly concave in k . The optimal scale of production is uniquely identified.

Note that the effect of an increase in $\bar{\varepsilon}$ on the firm's choice of k is ambiguous. The increase in $\bar{\varepsilon}$ raises the share of firm earnings which are paid out as tax-deductible interest payments. This reduces the effective marginal tax rate and encourages investment. The downside of an increase in $\bar{\varepsilon}$ is that expected default costs rise. This reduces the bond price and raises the spread. Shareholders have to inject more equity for a given amount of expected dividends. This discourages investment.

A first order condition for an optimal choice of $\bar{\varepsilon}$ is:

$$\bar{\varepsilon}: \quad [1 - \Phi(\bar{\varepsilon})] \left[(1 + c) \frac{\partial \tilde{b}}{\partial \bar{\varepsilon}} - (1 - \tau)k \right] - \varphi(\bar{\varepsilon})(1 + c)(\tilde{b} - b) = 0. \quad (15)$$

If default is avoided, an increase in $\bar{\varepsilon}$ reduces the expected amount of taxable dividend payments by $(1 - \tau)k$ and increases the payments to bond holders by $(1 + c) \frac{\partial \tilde{b}}{\partial \bar{\varepsilon}}$. This raises the share of firm earnings which is exempt from taxation. This benefit is measured by the term in squared brackets:

$$(1 + c) \frac{\partial \tilde{b}}{\partial \bar{\varepsilon}} - (1 - \tau)k = (1 - \tau)k \frac{\tau c}{1 + (1 - \tau)c} \quad (16)$$

This term is always positive. It is linear in k and increasing in the coupon c . The benefit is weighted with the likelihood that default is avoided, $1 - \Phi(\bar{\varepsilon})$. This likelihood is falling in $\bar{\varepsilon}$ which is captured by the second term of the first order condition (15) above. Given $\xi = 1$, bond holders lose the full amount of $(1 + c)\tilde{b}$ in case of default. Through creditors' break-even condition, the firm internalizes only the part of this loss which is borne by the buyers of newly issued bonds $(1 + c)(\tilde{b} - b)$. The firm disregards that by selling additional bonds it *dilutes* the value of previously issued bonds. The optimal value of $\bar{\varepsilon}$ is pinned down by the trade-off between the tax advantage of debt and the part of the expected costs of default which is internalized by the firm.

4.4.3. Cyclical Properties

We are interested in how the default rate and the credit spread move over the business cycle. High values of z create a boom period and low values of z a recession. Throughout this section, we assume that the firm's problem described above has an interior solution.

Proposition 4.1. *The default rate and the credit spread are falling in z if and only if $b > 0$. Without long-term debt ($b = 0$), the default rate, the credit spread and leverage are all constant in z .*

Proofs can be found in the appendix. A necessary condition for the result that the default rate, the credit spread, and leverage are all constant in z if $b = 0$ is the property of $f(k)$ that its marginal product is a constant fraction of its average product. The

marginal product $f'(k)$ appears in the first order condition associated to k , the average product $\frac{f(k)}{k}$ appears in the first order condition associated to $\bar{\varepsilon}$. The scale of production k responds to changes in z but the ratio between the marginal product and the average product remains constant. In the proof we show that this implies that the firm's choice of $\bar{\varepsilon}$ is independent of z .

Leverage is given by

$$\frac{\tilde{b}}{k} = \frac{1 + (1 - \tau) \left[z \frac{f(k)}{k} - \delta + \bar{\varepsilon} \right]}{1 + (1 - \tau)c}. \quad (17)$$

If $b = 0$, $\bar{\varepsilon}$ is constant in z . The cyclicity of leverage depends on average output per capital $z \frac{f(k)}{k}$. In the proof we show that the firm optimally adjusts k to keep $z \frac{f(k)}{k}$ constant.

The introduction of long-term debt profoundly affects the cyclical behavior of the default rate, the credit spread, and leverage in this economy. In the absence of outstanding long-term debt, changes in aggregate revenue productivity z leave the default rate, the credit spread, and leverage unaffected. But if the firm inherits some previously issued long-term debt, a drop in z increases the default rate and the credit spread.

Compensation Payments. Why does the introduction of long-term debt change the cyclical behavior of the firm? Consider the total value of previously issued bonds:

$$pb = \frac{1 - \Phi(\bar{\varepsilon})}{1 + r} b(1 + c). \quad (18)$$

This value decreases if the firm's current actions cause $\bar{\varepsilon}$ to rise. The firm takes creditors' break-even constraint into account when it chooses k and $\bar{\varepsilon}$. But only the value of newly issued bonds $\tilde{b} - b$ enters the firm's objective. The firm ignores changes in the value of previously issued bonds pb .

To see why this matters for the cyclical behavior of default, consider an alternative scenario in which the firm needs to compensate the owners of previously issued bonds if pb deviates from some reference value M . When the firm chooses equity and debt in period 0, it needs to pay a compensation $M - pb$.¹⁰ Subtracting the term $M - pb$ from the firm's objective yields a new maximization problem:

$$\max_{k, \bar{\varepsilon}} -k - M + \frac{1}{1 + r} [1 - \Phi(\bar{\varepsilon})] (1 + c) \tilde{b} + \frac{1}{1 + r} \int_{\bar{\varepsilon}}^{\infty} (1 - \tau) k [\varepsilon - \bar{\varepsilon}] \varphi(\varepsilon) d\varepsilon \quad (19)$$

$$\text{subject to: } \tilde{b} = \frac{k + (1 - \tau) [z f(k) - \delta k + \bar{\varepsilon} k]}{1 + (1 - \tau)c}.$$

Now the firm not only internalizes the value of newly issued bonds $\tilde{b} - b$, but the value of the entire stock of debt \tilde{b} , old and new bonds alike. From this problem's first order

¹⁰In models of long-term debt and sovereign default, this method is used by Chatterjee and Eyigungor (2012) and Hatchondo et al. (2016).

conditions, we can derive the following result.

Proposition 4.2. *If the firm must compensate creditors for changes in the value of previously issued bonds, the default rate, the credit spread, and leverage are all constant in z for any value of $b \geq 0$.*

It follows from Proposition 4.2 that the result stated in Proposition 4.1 is entirely due to the fact that the firm does not internalize the effect of its actions on the value of bonds issued in the past. An increase of $\bar{\varepsilon}$ through the sale of additional bonds dilutes the value of previously issued bonds. Shareholders ignore this effect. They take the increase of $\bar{\varepsilon}$ into account only as far as it reduces the value of newly issued bonds $p(\tilde{b} - b)$. This disregard of a part of the total default costs increases the firm's choice of $\bar{\varepsilon}$ for any given value of z . Debt dilution raises the default rate at each point of the business cycle.

Cyclical Debt Dilution. Proposition 4.1 states that the default rate is high if z is low if and only if $b > 0$. So why is it that debt dilution causes the default rate to move over the business cycle? We know from equation (15) that the benefit of a marginal increase in $\bar{\varepsilon}$ is proportional to k . The cost is proportional to $\tilde{b} - b$. If $b = 0$, leverage $\frac{\tilde{b}}{k}$ is constant in z . This implies that the cost of a marginal increase in $\bar{\varepsilon}$ is proportional to k as well. If $b > 0$, the firm still enjoys all the benefits of an increase in $\bar{\varepsilon}$ but it ignores a fraction $\frac{b}{k}$ of the associated costs. If k is large relative to b , this effect is small. But if k is small relative to b , the firm disregards a larger fraction of the total costs of an increase in $\bar{\varepsilon}$ and its choice of $\bar{\varepsilon}$ will be higher. Since k is low if z is low and b cannot respond to changes in z , $\frac{b}{k}$ is high if z is low. Debt dilution causes $\bar{\varepsilon}$ to increase. The default rate becomes counter-cyclical.

Debt Overhang. From the proofs of Proposition 4.1 and 4.2, an additional result follows which characterizes the role of outstanding long-term debt b .

Lemma 4.3. *The default rate and the credit spread are increasing in $b \geq 0$. If the firm needs to compensate creditors for changes in the value of previously issued bonds, the default rate, the credit spread, and leverage are all constant in $b \geq 0$.*

The intuition for this result is clear. The higher the fraction of the total potential costs of default which the firm disregards, the higher is the firm's choice of $\bar{\varepsilon}$. Once the firm is forced to internalize the effect of its actions on the value of outstanding long-term debt, this mechanism is absent. Gomes et al. (2016) find a closely related result. Just like in our model, firms are restricted to time-consistent policies. A negative shock to inflation increases the real value of outstanding long-term debt. Firms respond by choosing higher leverage and a higher risk of default. Leverage and default risk remain elevated until the real value of outstanding long-term debt has returned to its long-run mean. Viewed through the lens of our model, this phenomenon of "sticky leverage" can be understood as a manifestation of debt dilution. With nominal debt, a drop in the price level corresponds to a persistent increase of b in our model. Lemma 4.3 states that in the presence of debt dilution the default rate is an increasing function of b .

5. Dynamic Model

Now that we have studied the cyclical role of debt dilution in a two-period economy, we use these results to understand debt dilution in a fully dynamic model with a maturity choice. Firms can choose between selling short-term bonds and long-term bonds. Long-term debt is unattractive because it gives rise to debt dilution in the future. Short-term debt has the disadvantage that its entire amount needs to be rolled-over each period. This is costly because of a transaction cost on the bond market.

5.1. Setup

There is a unit mass of firms. As in the two-period economy, a firm i uses capital k_{it} to produce output y_{it} using a technology with diminishing returns:

$$y_{it} = z_t k_{it}^\alpha, \quad \alpha \in (0, 1). \quad (20)$$

Earnings before interest and taxes are given as:

$$z_t k_{it}^\alpha - \delta k_{it} + \varepsilon_{it} k_{it}. \quad (21)$$

The firm-specific earnings shock ε_{it} is i.i.d. and follows a probability distribution $\varphi(\varepsilon)$ with mean zero and standard deviation σ_ε .

In contrast to the two-period economy, the firm can now choose between two debt instruments of different maturity. Short-term debt and long-term debt are of equal seniority.

Definition: Short-term Debt. A short-term bond issued at the end of period t is a promise to pay one unit of the numéraire good together with a fixed coupon payment c in period $t + 1$. The quantity of short-term bonds sold by firm i is \tilde{b}_{it}^S .

Definition: Long-term Debt. A long-term bond issued at the end of period t is a promise to pay a fixed coupon payment c in period $t + 1$. In addition, the firm repays a fraction γ of the principal. In period $t + 2$, a fraction $1 - \gamma$ of the bond remains outstanding. Creditors receive a coupon payment $(1 - \gamma)c$. In addition, the firm repays a fraction of the remaining principal: $(1 - \gamma)\gamma$. In this manner, payments geometrically decay over time. The maturity parameter γ controls the speed of decay. The quantity of long-term bonds is \tilde{b}_{it}^L .

This computationally highly tractable specification of long-term debt goes back to Leland (1994).¹¹

¹¹The Macaulay duration μ of this long-term bond is:

$$\mu = \frac{1}{p_r^L} \sum_{j=1}^{\infty} j (1 - \gamma)^{j-1} \frac{c + \gamma}{(1 + r)^j} = \frac{c + \gamma}{p_r^L} \frac{1 + r}{(\gamma + r)^2}, \quad (22)$$

Definition: Transaction cost. The firm pays an amount η for each unit of short-term bonds sold and each unit of long-term bonds sold or repurchased:

$$H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it}^L) = \eta(\tilde{b}_{it}^S + |\tilde{b}_{it}^L - b_{it}^L|), \quad (25)$$

where b_{it}^L is the stock of outstanding long-term bonds.

Firm net worth after production is:

$$q_{it} = k_{it} - \tilde{b}_{it}^S - \gamma \tilde{b}_{it}^L + (1 - \tau)[z_t k_{it}^\alpha - \delta k_{it} + \varepsilon_{it} k_{it} - c(\tilde{b}_{it}^S + \tilde{b}_{it}^L)]. \quad (26)$$

The firm raises capital by injecting equity and from selling short- and long-term bonds:

$$k_{it} = e_{it} + p_{it}^S \tilde{b}_{it}^S + p_{it}^L (\tilde{b}_{it}^L - b_{it}^L) - H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it}^L). \quad (27)$$

Dividends are given by

$$d_{it} = q_{it} - e_{it+1}. \quad (28)$$

Definition: Limited Liability. Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. A fixed fraction ξ of the firm's assets is lost in this case. After default, shareholders are free to start a new business with zero equity and zero debt.

Timing

End of period $t - 1$: Aggregate revenue productivity z_t is realized. Firm i has an amount b_{it}^L of long-term debt outstanding. Given z_t and b_{it}^L , the firm chooses next period's book value of equity e_{it} . It also decides how to adjust its level of long-term debt \tilde{b}_{it}^L and how many short-term bonds \tilde{b}_{it}^S to sell. This determines next period's stock of capital k_{it} .

Beginning of period t : The firm draws the realization ε_{it} . This determines firm earnings. The firm decides whether to default. If it decides not to default, it pays corporate income tax on earnings net of depreciation and coupon payments. This leaves the firm with net worth q_{it} . Next period's amount of long-term debt is $b_{it+1}^L = (1 - \gamma)\tilde{b}_{it}^L$.

where p_r^L is the price of a riskless long-term bond:

$$p_r^L = \sum_{j=1}^{\infty} (1 - \gamma)^{j-1} \frac{c + \gamma}{(1 + r)^j} = \frac{c + \gamma}{r + \gamma}. \quad (23)$$

It follows for the Macaulay duration:

$$\mu = \frac{1 + r}{\gamma + r}. \quad (24)$$

5.2. Firm Problem

The firm maximizes shareholder value. As in the two-period economy, there is a unique threshold realization $\bar{\varepsilon}_{it}$ which decides whether shareholders prefer to default:

$$\bar{\varepsilon}_{it} : \quad k_{it} - \tilde{b}_{it}^S - \gamma \tilde{b}_{it}^L + (1 - \tau)[z_t k_{it}^\alpha - \delta k_{it} + \bar{\varepsilon}_{it} k_{it} - c(\tilde{b}_{it}^S + \tilde{b}_{it}^L)] \\ + \mathbb{E}_{z_{t+1}|z_t} V\left((1 - \gamma)\tilde{b}_{it}^L, z_{t+1}\right) = \mathbb{E}_{z_{t+1}|z_t} V\left(0, z_{t+1}\right), \quad (29)$$

where $V(b_{it+1}^L, z_{t+1})$ denotes the end-of-period- t stock market value of a firm with long-term debt b_{it+1}^L when aggregate revenue productivity is z_{t+1} . If ε_{it} turns out to be smaller than $\bar{\varepsilon}_{it}$, shareholders optimally choose to stop paying their liabilities and default.

We assume that the firm has no ability to commit to future actions. The firm understands the decision problem it will face at future points in time. It must take its own future behavior as given when deciding about its policy today. We restrict attention to strategies which are functions of the current state of the firm. That is, we are studying the Markov Perfect equilibrium of the game which the firm plays against its future selves.

The firm maximizes the discounted sum of future dividend payments. For ease of exposition, we drop firm- and time-subscripts from now on. The state of the firm is $s = (b, z)$. The firm chooses a policy vector $\phi(s) = (k, e', \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon})$ as a solution to the problem

$$V(b, z) = \max_{k, e', \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}} \quad - e' + \frac{1}{1 + r} \left[\int_{\bar{\varepsilon}}^{\infty} \left[q' + \mathbb{E}_{z'|z} V((1 - \gamma)\tilde{b}^L, z') \right] \varphi(\varepsilon) d\varepsilon \right. \\ \left. + \Phi(\bar{\varepsilon}) \mathbb{E}_{z'|z} V(0, z') \right] \quad (30)$$

$$\text{subject to: } \quad q' = k - \tilde{b}^S - \gamma \tilde{b}^L + (1 - \tau)[z k^\alpha - \delta k + \varepsilon k - c(\tilde{b}^S + \tilde{b}^L)]$$

$$\bar{\varepsilon} : \quad q' + \mathbb{E}_{z'|z} V((1 - \gamma)\tilde{b}^L, z') = \mathbb{E}_{z'|z} V(0, z')$$

$$k = e' + p^S \tilde{b}^S + p^L (\tilde{b}^L - b) - H(\tilde{b}^S, \tilde{b}^L, b).$$

The two bond prices p^S and p^L depend on the firm's behavior.

5.3. Creditors' Problem

Once the firm stops paying its liabilities and defaults, the value of the firm's assets is

$$\tilde{q}(k, \bar{\varepsilon}, z) \equiv k + (1 - \tau)[z k^\alpha - \delta k + \bar{\varepsilon} k]. \quad (31)$$

At this point, creditors liquidate the firm's assets and receive $(1 - \xi)\tilde{q}(k, \bar{\varepsilon}, z)$. There are two bond prices: p^S and p^L . On expectation, risk-neutral creditors' break even on both

types of bonds:

$$p^S(s, \phi(s)) = \frac{1}{1+r} \left[[1 - \Phi(\bar{\varepsilon})] (1+c) + \Phi(\bar{\varepsilon}) \frac{(1-\xi)\tilde{q}(s, \phi(s))}{\tilde{b}^S + \tilde{b}^L} \right], \quad \text{and} \quad (32)$$

$$p^L(s, \phi(s)) = \frac{1}{1+r} \left[[1 - \Phi(\bar{\varepsilon})] \left(\gamma + c + (1-\gamma) \mathbb{E}_{z'|z} p^L(s', \phi(s')) \right) + \Phi(\bar{\varepsilon}) \frac{(1-\xi)\tilde{q}(s, \phi(s))}{\tilde{b}^S + \tilde{b}^L} \right]. \quad (33)$$

Definition: Credit Spreads. As in the two-period economy, the spread on short-term bonds is:

$$1 + r^S - (1+r) = \frac{1+c}{p^S} - (1+r). \quad (34)$$

Assuming a constant price p^L , the return on risky long-term bonds is:

$$1 + r^L = \frac{c + \gamma + (1-\gamma)p^L}{p^L} = \frac{c + \gamma}{p^L} + 1 - \gamma. \quad (35)$$

It follows for the long-term spread:

$$1 + r^L - (1+r) = \frac{c + \gamma}{p^L} + 1 - \gamma - (1+r). \quad (36)$$

5.4. Equilibrium

As in the two-period economy, we solve for the partial equilibrium allocation given the exogenous and fixed discount rate $\frac{1}{1+r}$. In equilibrium, firms maximize shareholder value subject to creditors' two break-even conditions.

Recursive Markov Equilibrium. A recursive equilibrium for this economy is (i) a set of policy functions for capital $k(b, z)$, equity $e'(b, z)$, short-term debt $\tilde{b}^S(b, z)$, long-term debt $\tilde{b}^L(b, z)$, and a default threshold $\bar{\varepsilon}(b, z)$, and (ii) price functions for short-term debt $p^S(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, z)$ and long-term debt $p^L(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, z)$, such that the following conditions hold:

1. Taking the bond price functions $p^S(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, z)$ and $p^L(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, z)$ as given, the policy functions $k(b, z)$, $e'(b, z)$, $\tilde{b}^S(b, z)$, $\tilde{b}^L(b, z)$, and $\bar{\varepsilon}(b, z)$ solve the firm problem (30).
2. The bond price functions $p^S(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, z)$ and $p^L(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, z)$ satisfy creditors' break-even conditions (32) and (33).

There is a time-inconsistency problem which enters through the price of long-term debt $p^L(s, \phi(s))$. Through next period's price $p^L(s', \phi(s'))$, this price depends on the firm's behavior in the future. Creditors anticipate future debt dilution. This depresses today's price of long-term debt. The firm would like to commit to internalize all future

expected costs of default on bonds issued today. This would raise the bond price today and increase shareholder value. From the point of view of the firm today, the future policy $\phi(s')$ is not chosen optimally. Lack of commitment means that the only way in which the firm can limit debt dilution is through the future state variable $(1 - \gamma)\tilde{b}^L$.

Firms differ with respect to the amount of long-term debt outstanding. Some firms have defaulted in the past and started anew with zero debt. Given a distribution of firms over long-term debt outstanding b , aggregate variables are constructed as a weighted average of firm policies.

5.4.1. Consolidated Problem

As in the two-period economy, it is useful to simplify the problem. The consolidated problem will allow us to numerically compute equilibrium bond prices and firm policies in a single step. We define the share of long-term debt as

$$m = \frac{\tilde{b}^L}{\tilde{b}^S + \tilde{b}^L}. \quad (37)$$

Given m we can back out the amount of short-term bonds:

$$\tilde{b}^S = \frac{1 - m}{m} \tilde{b}^L. \quad (38)$$

Using the definition of $\bar{\varepsilon}$, the value of \tilde{b}^L can be expressed as:

$$\tilde{b}^L = G(k, \bar{\varepsilon}, m, z). \quad (39)$$

From $k = e' + p^S \tilde{b}^S + p^L (\tilde{b}^L - b) - H(\tilde{b}^S, \tilde{b}^L, b)$, we derive:

$$e' = k - p^S \tilde{b}^S - p^L (\tilde{b}^L - b) + H(\tilde{b}^S, \tilde{b}^L, b). \quad (40)$$

The firm objective becomes

$$\begin{aligned} & -k + p^S \tilde{b}^S + p^L (\tilde{b}^L - b) - H(\tilde{b}^S, \tilde{b}^L, b) \\ & + \frac{1}{1 + r} \left[\int_{\bar{\varepsilon}}^{\infty} \left[k - \tilde{b}^S - \gamma \tilde{b}^L + (1 - \tau)[zk^\alpha - \delta k + \varepsilon k - c(\tilde{b}^S + \tilde{b}^L)] \right. \right. \\ & \quad \left. \left. + \mathbb{E}_{z'|z} V((1 - \gamma)\tilde{b}^L, z') \right] \varphi(\varepsilon) d\varepsilon + \Phi(\bar{\varepsilon}) \mathbb{E}_{z'|z} V(0, z') \right]. \quad (41) \end{aligned}$$

We have expressed \tilde{b}^S and \tilde{b}^L as functions of the choice variables k , $\bar{\varepsilon}$, m , and the aggregate state z . Using creditors' zero-profit condition, this allows us to pin down the short-term bond price $p^S(s, \phi(s))$ as a function of the same three choice variables and the aggregate state. Similarly, the long-term bond price $p^L(s, \phi(s))$ can be expressed as a function of k , $\bar{\varepsilon}$, m , z , together with tomorrow's price $p^L(s', \phi(s'))$. Given tomorrow's price function of long-term debt $p^L(s', \phi(s'))$ and a probability distribution for the ex-

ogenous state z , both bond prices $p^S(s, \phi(s))$ and $p^L(s, \phi(s))$ are pinned down by the firm's choice of k , $\bar{\varepsilon}$, and m .

The firm's problem is simply to choose values for k , $\bar{\varepsilon}$, and m which maximize its objective function (41) taking as given tomorrow's price function for long-term debt $p^L(s', \phi(s'))$ and the firm's value function $V(b', z')$. The solution to this consolidated problem computes the equilibrium bond prices, $p^S(s, \phi(s))$ and $p^L(s, \phi(s))$, and today's firm policy, k , $\bar{\varepsilon}$, m , in a single step.

5.5. Quantitative Analysis

The firm's choice between short-term and long-term debt can only be studied in a fully dynamic economy. That is why we turn to a quantitative analysis.

5.5.1. Solution Method

We solve the model numerically using value function iteration and interpolation. We follow the literature on sovereign default with long-term debt in that we solve for the equilibrium allocation of a finite-horizon economy. Starting from the final date, we iterate backward in time until the firm's value function and the two bond prices have converged. We then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium functions.¹²

Common practice in the literature on long-term debt is to compute the complete bond price schedules for all possible states and actions: $p^S(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, z)$ and $p^L(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, z)$. These price schedules are then used to compute the optimal policy. In Section 5.4.1 we have re-formulated the firm problem in a way which expresses equilibrium bond prices as a function of today's choice variables. Given the firm's future policy and a probability distribution for the exogenous state z , both bond prices are pinned down by the firm's choices today. This allows us to use the consolidated problem to compute equilibrium bond prices and today's firm policy in a single step. This reduces the number of necessary computations and allows for a faster and more precise solution.

5.5.2. Parametrization

We choose parameter values in order to replicate a number of key statistics from the U.S. for the time period 1984 - 2015, presented in Table 4. In the data, short-term liabilities have term to maturity of up to one year. We therefore choose the model period to be one year. Model counterparts of targeted empirical moments are summarized in Table 5. If $c = r$, the price of a riskless short-term bond and a riskless long-term bond in our model are both equal to one.

A key parameter in the model is γ . This parameter controls the speed of decay in payments on the long-term bond. We choose its value to match the average Macaulay duration of 6.5 years reported by Gilchrist and Zakrajšek (2012) for a sample of 5,982

¹²This solution method is used for instance by Hatchondo and Martinez (2009).

senior unsecured bonds issued by U.S. non-financial firms. Table 6 reports our choice for the full set of parameter values.

	Mean
1984-2012:	
Default Rate (yearly)	0.72%
1984-2015:	
Leverage: Liabilities / Assets	45.7%
Long-term Liabilities Share	67.1%
1973-2010:	
Macaulay Duration	6.5 years
1997-2015:	
Spread 1-3 years	1.32%
Spread 3-5 years	1.46%
Spread 5-7 years	1.69%
Spread 7-10 years	1.70%
Spread 10-15 years	1.71%
Spread 15 years and more	1.82%

Table 4: Empirical Moments

Note: Default rates are from Giesecke et al. (2014). This is the ratio of 'total par value of outstanding non-financial corporate bonds which are in default' over 'total par value of outstanding non-financial corporate bonds'. Total Assets, Short-term Liabilities, and Total Liabilities of non-financial corporate firms are from the Flow of Funds. Duration is from Gilchrist and Zakrajšek (2012). Data on Corporate Bond Spreads is from the St. Louis Fed Data Base (FRED). This is the Bank-of-America Merrill-Lynch US Corporate Option-Adjusted Spread using bonds of investment grade or higher.

Aggregate revenue productivity z_t follows an AR(1)-process:

$$z_t = \rho z_{t-1} + \epsilon_z, \quad (42)$$

where ϵ_z is white noise with standard deviation σ_z . The firm-specific earnings shock is Normal with zero mean and standard deviation σ_ϵ . We choose values for ρ and σ_z in order to generate an empirically plausible time path of GDP.¹³

We set the yearly rate of return on a riskless asset to $r = 3.09\%$. The parameter values of α and τ are taken from Covas and Den Haan (2012). The parameter δ is meant to account for several types of costs which are absent from our model, e.g. wages. Our choice generates a mean capital-output ratio of about 2.4.

The values of ξ , η , and σ_ϵ are chosen to match three empirical moments: leverage, the share of long-term liabilities, and bond spreads. We use two untargeted moments to verify that our model generates reasonable statistics. The first one is the average compensation paid to investment banks for bond placements. Altinkılıç and Hansen

¹³Using yearly Flow of Funds data from 1984-2015, we estimate an AR(1) process for the detrended log real GDP. We use the Hodrick-Prescott filter with a smoothing parameter of 6.25 as recommended by Ravn and Uhlig (2002) for yearly data. Our estimates yield an AR(1) coefficient of 0.41 with standard deviation in the error term of 0.01. Our parametrization targets these estimates.

Default Rate	$\Phi(\bar{\varepsilon})$
Total Liabilities	$D \equiv \frac{1+c}{1+r} \tilde{b}^S + \frac{\gamma+c}{\gamma+r} \tilde{b}^L$
Leverage: Liabilities / Assets	$\frac{D}{k}$
Long-term Liabilities Share	$\frac{1}{D} \frac{\gamma+c}{\gamma+r} \tilde{b}^L$
Macaulay Duration	$\frac{1+r}{\gamma+r}$
Short-term Spread	$\frac{1+c}{p^S} - (1+r)$
Long-term Spread	$\frac{c+\gamma}{p^L} + 1 - \gamma - (1+r)$
Term Structure	$\frac{c+\gamma}{p^L} + 1 - \gamma - \frac{1+c}{p^S}$

Table 5: Key Variables

(2000) report this value to be 1.09% of the proceeds. The corresponding value in our parametrized model is remarkably close: 0.84%. This suggests that our parameter choice for the transaction cost on the bond market η is reasonable. The second untargeted moment is the debt recovery rate in bankruptcy. On average, our parametrized model generates a recovery rate $(1 - \xi)\tilde{q}[\tilde{b}^S + \tilde{b}^L]^{-1}$ of 54%. This is broadly in line with the empirical evidence provided by Bris, Welch, and Zhu (2006) who document a mean recovery rate of 69% for Chapter 11 re-organizations and 27% for Chapter 7 liquidations.

It is well known that empirical bond spreads are not fully explained by realized default risk.¹⁴ In our model, bonds spreads are driven exclusively by default risk. This means that we have to decide whether we want our model to match the default rate and generate unrealistically low bond spreads, or if want to match bond spreads at the cost of generating unrealistically high default rates. In our model, the bond price schedule is key to understanding firm behavior. We therefore choose to match bond spreads rather than the default rate.

r	0.0309	γ	0.1277	ξ	0.45	σ_z	0.0035
α	0.7	c	r	η	0.0082	σ_ε	0.525
δ	0.25	τ	0.296	ρ	0.77		

Table 6: Parametrization

¹⁴See for instance Elton, Gruber, Agrawal, and Mann (2001).

5.5.3. Cyclical Properties

We now present numerical results on the response of real and financial variables to innovations in aggregate revenue productivity z . We focus on the model's ability to generate the counter-cyclical behavior of default rates, bond spreads, leverage, and debt maturity, as well as the pro-cyclical term structure of bond spreads.

Policy Functions. Figures 4 and 5 show firms' equilibrium policies as functions of the exogenous state z and the stock of outstanding long-term debt b . We see that the analytical results from Proposition 4.1 and 4.3 continue to hold in the fully dynamic economy: As the upper four panels of Figure 5 show, the default rate, leverage, and bond spreads are all constant in z as long as $b = 0$. For positive levels of outstanding debt, the default rate, leverage, and bond spreads are falling in z . All four variables are increasing in b . The analytical results from Section 4 allow us to attribute these results to debt dilution.¹⁵

In Figure 4, we see that firms choose higher values of capital k and output y if z is high. This is because the expected marginal return on capital is increasing in z . Also note that these variables are falling in b . As the default rate is increasing in b (because of debt dilution), the expected return on capital is falling.

If the stock of outstanding long-term debt is high, the firm will choose high levels of both short-term and long-term debt. In the words of Gomes et al. (2016), debt is "sticky". This result is due to debt dilution. As b increases, the fraction of the total potential costs of default which is internalized by the firm falls. The firm's response is to accept a higher risk of default which translates into higher leverage. The dashed line in the middle panel on the right side of Figure 4 indicates stable points for long-term debt. In our parametrization, the stable choice of long-term debt \tilde{b}^L for non-defaulting firms lies around 5.6 which, given our choice of $\gamma = 0.1277$, implies a corresponding value for outstanding long-term debt $b = (1 - \gamma)\tilde{b}^L$ of around 4.9.

The way equity e' responds to a higher level of b depends on the relative strength of two forces. On the one hand, higher values of b and higher credit spreads push towards a higher value of e' for given values of k and debt. On the other hand, the fact that both short-term debt and long-term debt are increasing in b and that capital k is falling pushes in the opposite direction. Figure 4 shows the resulting equilibrium outcome. Equity is increasing for low values of b and falling for higher values of b .

Shareholder value (i.e. the firm's value function) is increasing in z and falling in b . If b is high, a large part of the total firm value has been promised to the holders of previously issued long-term debt and only a small part is left for shareholders.

The left panel in the bottom row of Figure 5 shows the term structure of bond spreads: the difference between the long-term and short-term bond spreads. For low values of b , creditors expect that the firm will increase the amount of long-term debt in the future.

¹⁵If we had not allowed firms to issue long-term debt, the default rate, leverage, and bond spreads would all be perfectly a-cyclical. In Appendix B.2 we show firms' equilibrium policies when only short-term debt is available.

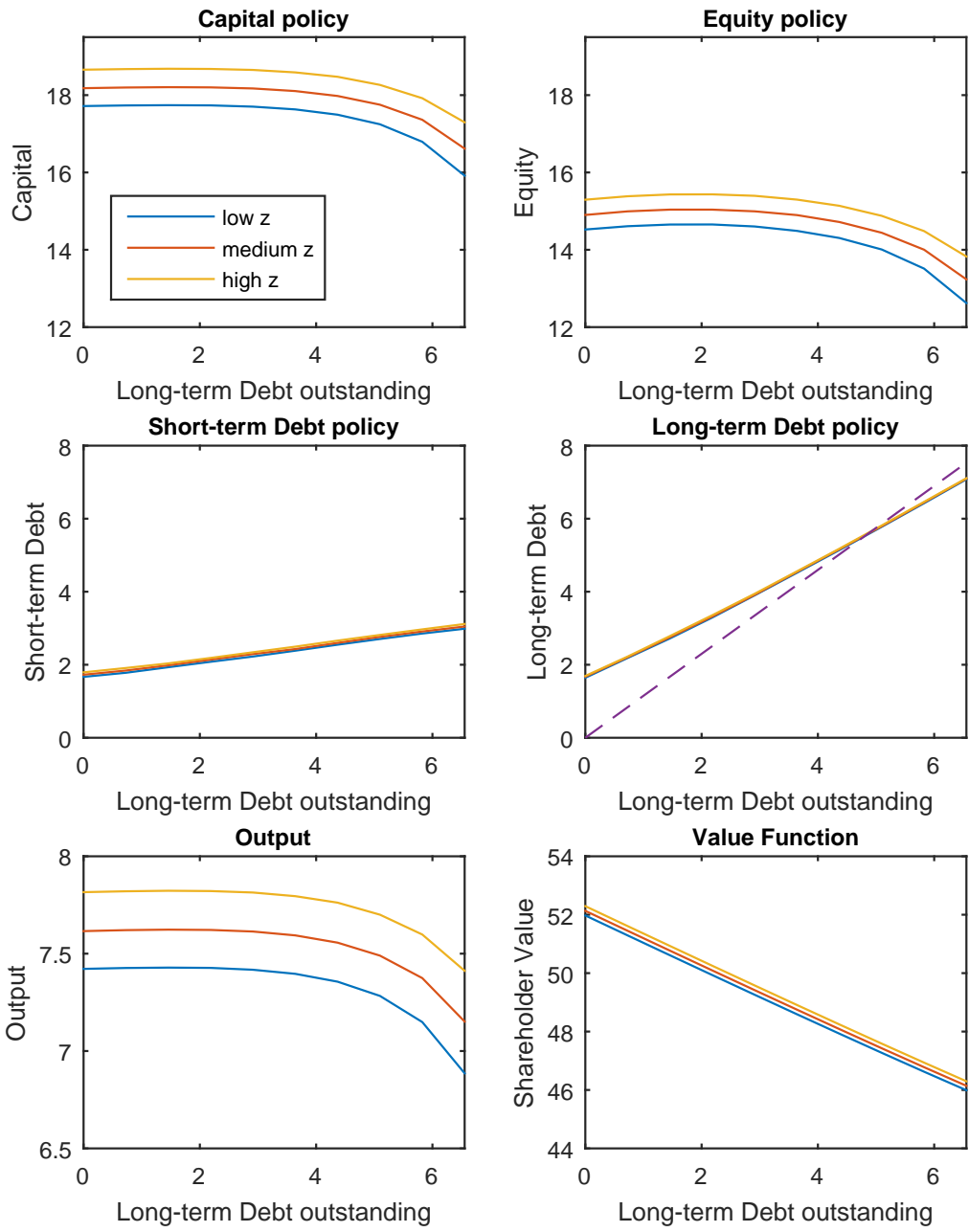


Figure 4: Policy Functions Part I.

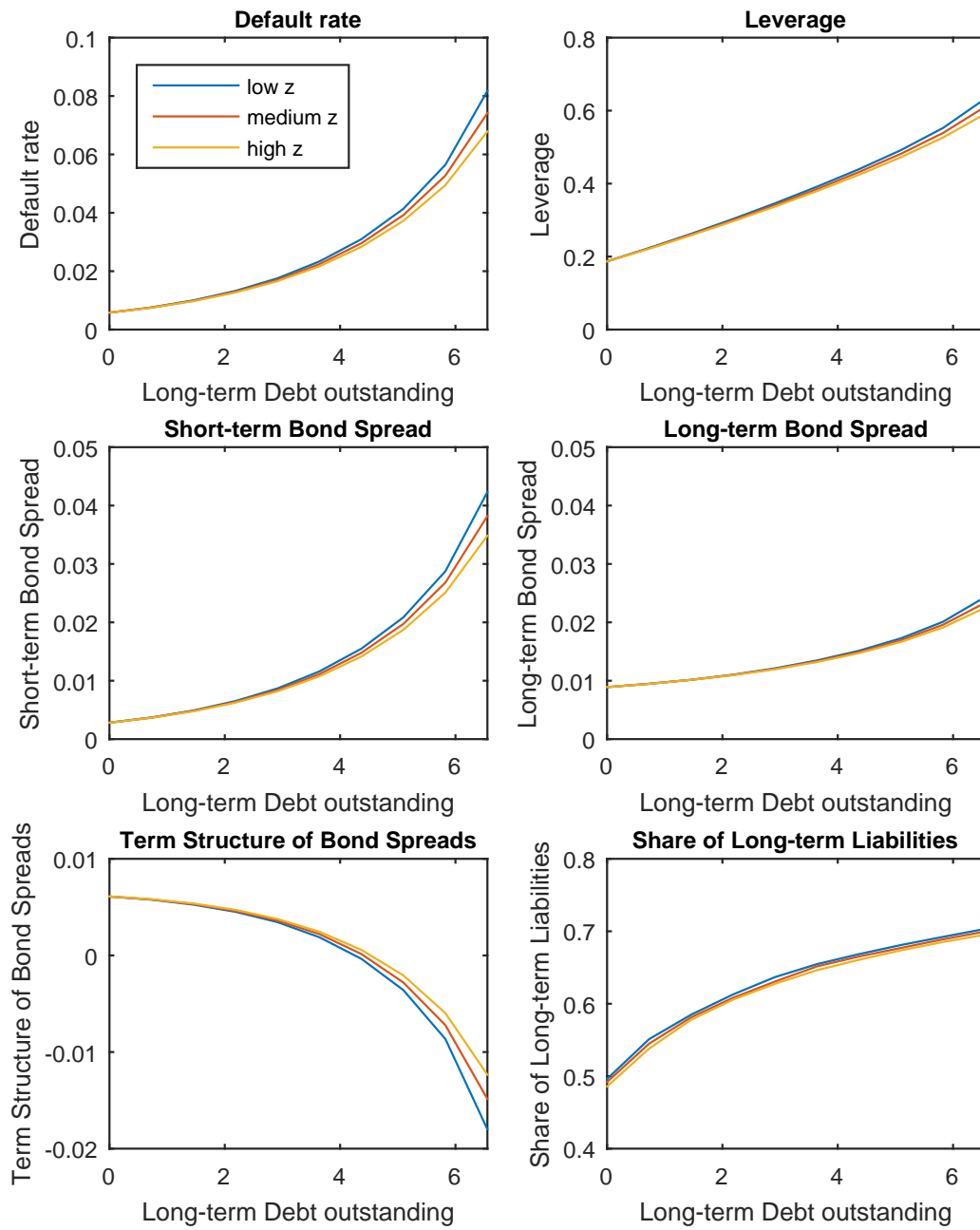


Figure 5: Policy Functions Part II.

Because of debt dilution, this will increase the risk of default. This affects the long-term bond more than the short-term bond, hence the term structure is positive for low values of b . If b is very high, creditors expect that the firm will lower the amount of long-term debt in the future. This will reduce the extent of future debt dilution. Again, this future reduction in default risk affects the long-term bond much more than the short-term bond. For this reason, the term structure is negative for high values of b .

Because of cyclical debt dilution, temporarily high values of z decrease the risk of default today. This strongly affects the short-term bond spread which depends exclusively on the risk of default today. The long-term bond spread depends on the risk of default in all future periods. Because of mean reversion, the long-term spread is much less affected by a temporary decline in the risk of default than the short-term spread. Therefore the short-term spread falls more strongly in z and the difference between the long-term spread and the short-term spread increases in z .

The right panel in bottom third row of Figure 5 shows that the firm's share of long-term debt is an increasing function of b . Evidently, the firm's time-inconsistency problem also affects its maturity choice. For a given amount of total debt, an increase in the long-term debt share reduces the amount of debt which needs to be rolled over each period. Because of the transaction cost on the bond market, this is beneficial. The cost of an increase in the long-term debt share is that debt dilution becomes more severe in the future. Part of this cost is internalized by the firm through a fall in the long-term bond price today. But another part of this cost is borne by the holders of outstanding long-term debt. The firm disregards this part. As a result, the firm chooses a higher long-term debt share if b is high.

Just like the default rate, leverage, and bond spreads, the long-term debt share is falling in z . The underlying mechanism is identical. As z is increasing, the ratio of newly issued debt to outstanding debt increases as well. This raises the share of the total value of debt which is internalized by the firm. The firm benefits more from a future reduction in debt dilution and therefore chooses to reduce the long-term debt share.

Firm Distribution. At each point in time, the population of firms is heterogeneous with respect to the amount of long-term debt outstanding b . Figure 6 shows three firm distributions. For three different levels of z , we plot the distribution which arises following a long sequence of identical realizations of z . For each given value of z , the majority of firms has not defaulted for a long time. These firms eventually find themselves near the stable values for outstanding long-term debt of around 4.9. Firms always choose a positive risk of default. A firm which defaulted continues to operate with $b = 0$. Debt dilution is not a concern for this firm. For this reason, the firm initially chooses low values of \tilde{b}^L . But with a positive amount of long-term debt outstanding, debt dilution induces the firm to take on more and more long-term debt over time, until the firm reaches a stable value.

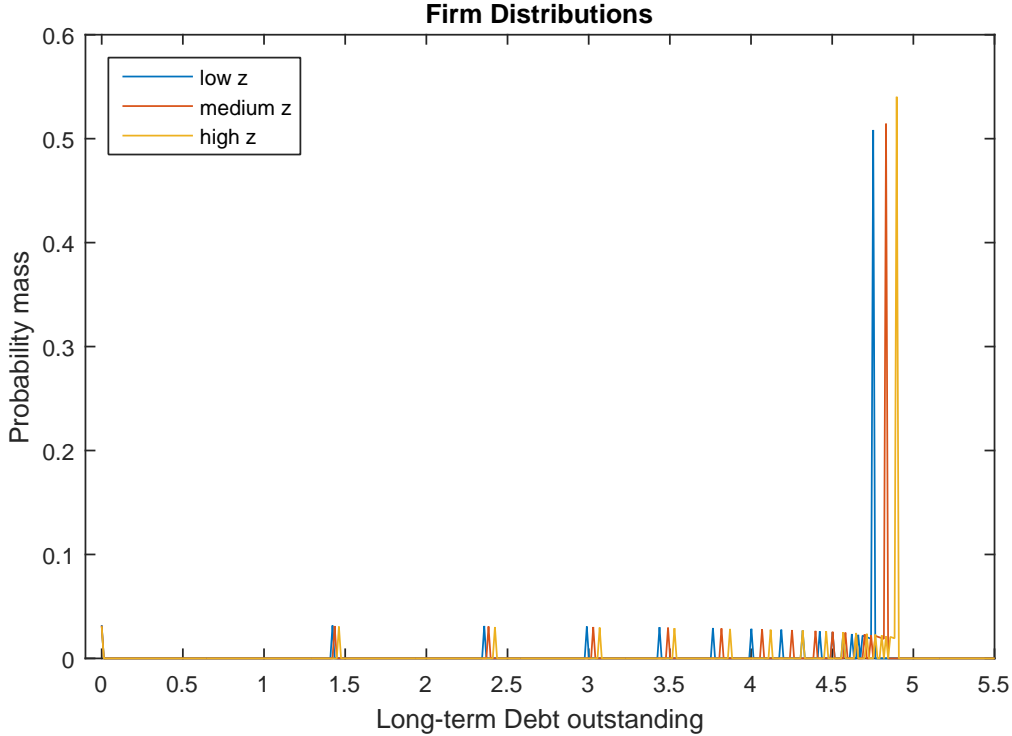


Figure 6: Firm Distributions.

Impulse Response Functions. The impulse response functions shown in Figure 7 confirm our observations from studying firms' equilibrium policy. We study the response of aggregate variables to a one-time drop in z . Aggregate variables are constructed as a weighted average of firm policies using the time-varying firm distribution over b . This distribution does not play an interesting role in this model economy. In the aggregate, leverage, the default rate, bond spreads, and the long-term debt share are all somewhat lower than for the median firm which finds itself close to the stable value of b . Apart from that, impulse response functions of the median firm are qualitatively identical to impulse response functions of aggregate variables.

Because of cyclical debt dilution, the default rate, leverage, bond spreads, and the long-term debt share all increase after a negative shock to aggregate revenue productivity z . Short-term spreads are more volatile than long-term spreads. For this reason, the term structure of bond spreads falls together with z . Debt dilution is crucial to replicate these empirical facts in this model economy. If only short-term debt was available, the default rate, leverage, and bond spreads would all be perfectly flat over the business cycle.¹⁶

Note that the unconditional level of long-term spreads is slightly below the level of short-term spreads. In case of default, short-term creditors lose a promised payment of

¹⁶In Appendix B.2 we show impulse response functions for an alternative economy where only short-term debt is available to firms.

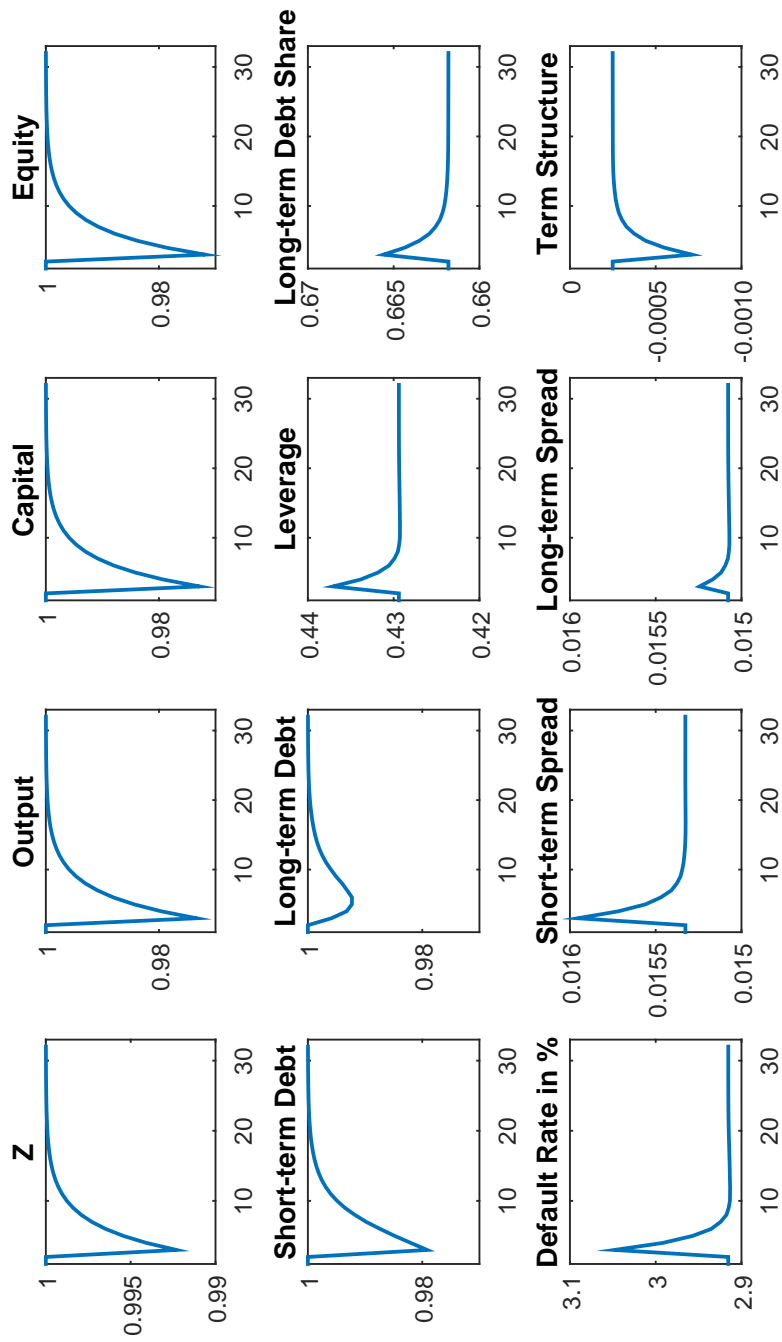


Figure 7: Impulse Response Functions.

$1 + c$. Long-term creditors lose $\gamma + c$ in addition to future payments of value $(1 - \gamma)p^L$. Since the equilibrium price of risky long-term debt p^L is smaller than 1, long-term creditors lose less than short-term creditors. However, both of them are entitled to an identical claim on the liquidation value of the firm's assets $(1 - \xi)\tilde{q}$. For this reason, long-term creditors demand a smaller average spread in our model than short-term creditors.¹⁷

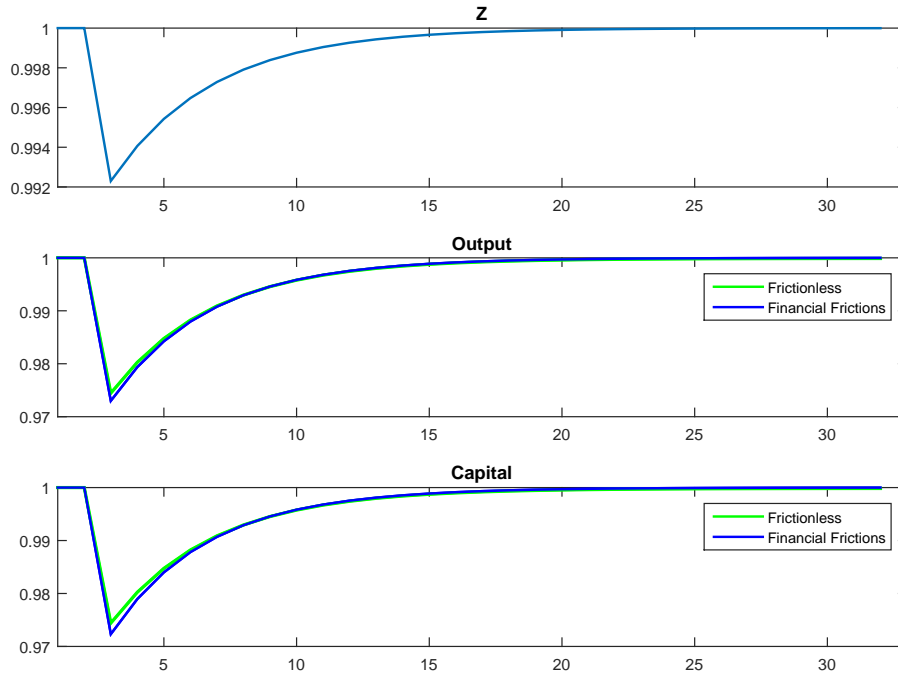


Figure 8: Amplification through Cyclical Debt Dilution.

Amplification. Figure 8 compares three of the impulse response function from Figure 7 with a model of frictionless capital markets and zero taxation. The first best level of capital is described by the following first order condition:

$$z\alpha k_{fl}^{\alpha-1} - \delta - r = 0 \quad \Leftrightarrow \quad k_{fl} = \left(\frac{z\alpha}{\delta + r} \right)^{\frac{1}{1-\alpha}}. \quad (43)$$

The responses of output and capital in our benchmark economy are slightly more pronounced than in the frictionless case. In response to the negative shock to z , output

¹⁷This preferential treatment of long-term bonds also influences firms' maturity choice. In our parametrization, the quantitative significance of this channel turns out to be small. Without the transaction cost on the bond market ($\eta = 0$), firms barely issue any long-term debt.

falls by 2.7% compared to 2.55% in the frictionless model. This implies an amplification of about 5%.¹⁸

Cyclical debt dilution causes credit spreads to increase during a downturn. Rising spreads feed back into low investment and less borrowing. This increases the ratio of previously issued debt to newly issued debt further, making debt dilution even more attractive. Cyclical debt dilution is a potentially powerful feedback loop between low investment and high credit spreads. Given our choice of parameter values, we find its quantitative significance in amplifying firms' response to fundamental shocks to be moderate. Different parametrizations, which generate a higher average rate of default, also generate stronger amplification.

5.5.4. Policy Exercises

Long-term debt allows firms to save transaction costs on the bond market, but it also creates a time-inconsistency problem. In order to understand the associated costs, we consider two policy experiments. First we study an outright ban of long-term debt. Second, we quantify the costs of debt dilution by comparing the results from our benchmark economy to an alternative economy in which long-term bond contracts feature a debt covenant which completely neutralizes debt dilution. We also refer to empirical evidence indicating that similar debt covenants are not used in practice.

Throughout this section, we continue to study a model economy with aggregate shocks to z . But for illustrative purposes, we assume that during our policy experiments the exogenous state z happens to stay at its unconditional mean. As a welfare criterion, we use the surplus created in the corporate sector:

$$W = \int_0^1 [y(i) - \delta k(i) - rk(i)] di, \quad (44)$$

where $y(i)$ and $k(i)$ are firm i 's choices of output and capital. This welfare criterion implicitly assumes that taxes, earnings shocks, bankruptcy costs, and the transaction cost on the bond market are all purely redistributive and are not a cost to society as a whole. We interpret the transaction cost on the bond market as a fee charged by investment banks for their services. Similarly, some part of bankruptcy costs consists of payments to lawyers and auditors. According to our welfare criterion, financial frictions matter only insofar as they distort the allocation of capital.¹⁹ To the extent that there are real social costs of bankruptcy, our welfare criterion provides a lower bound for the efficiency gap with respect to the first best economy without taxation and with frictionless capital markets.

¹⁸If firms are restricted to using only short-term debt as in Appendix B.2, there is no discernible difference between impulse response functions with and without financial frictions.

¹⁹We ignore the fact that lawyers and investment bankers could pursue different valuable activities in a first best economy.

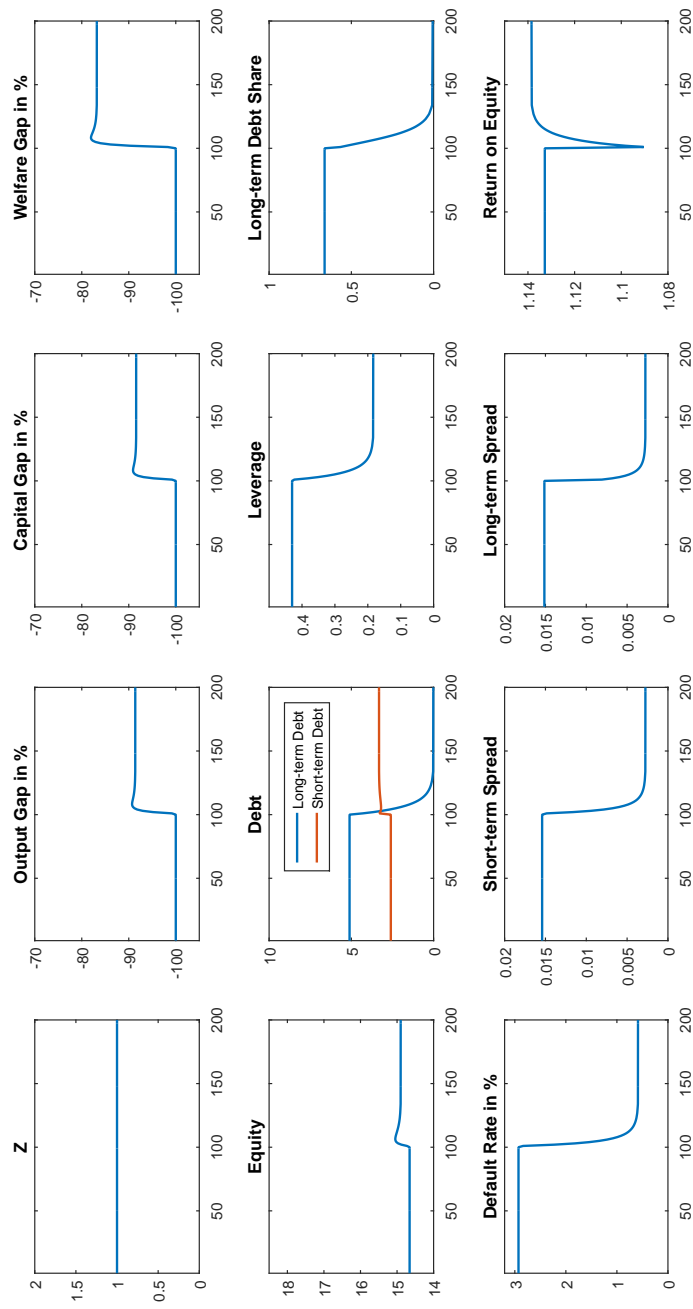


Figure 9: Ban of Long-term Debt.

Policy 1: Ban of Long-term Debt

A somewhat extreme policy experiment is an outright ban of long-term debt. In the model, we consider a constraint which forces firms to set $\tilde{b}^L = b$ from time t_p onwards. This constraint implies that firms continue to service outstanding long-term bonds, but they are unable to issue new ones. Over time, firms' stock of long-term debt converges towards zero. Up to time t_p , the ban of long-term is completely unexpected.

Figure 9 shows what happens in our model economy after the policy intervention at time $t_p = 100$. In the first period after the policy intervention, the default rate, leverage, and bond spreads fall only mildly. The reason is that initially the stock of long-term debt is still high. Debt dilution is still a concern. The firm makes up for the forced reduction in long-term debt by sharply increasing its stock of short-term debt. As the stock of long-term debt is gradually reduced, debt dilution becomes weaker. The default rate, leverage, and bond spreads all fall and converge to a lower level. The long-run level of the default rate is about 0.6% compared to almost 3% before the policy intervention. Leverage falls from 43% to 18.5%, credit spreads from 1.5% to 0.3%.

In the first row of Figure 9, we compare our benchmark economy with the first best allocation. In the first best economy with frictionless capital markets and zero taxes, the allocation of capital is undistorted. Output, capital, and welfare are all higher than in our benchmark economy. For each period, we calculate the gap in output, capital, and welfare between the two economies. This gap is always positive. We normalize it to 100% prior to the policy and track its evolution in response to the policy intervention in period t_p .

Output, capital, and welfare all increase in response to the ban of long-term debt. There are two opposing effects at work. As leverage is falling, the effective tax rate on capital increases. This lowers the marginal return on capital. On the other hand, long-term debt gradually disappears following the policy change. Firms respond by reducing leverage. This allows creditors to charge lower short-term bond spreads, rendering investment more profitable for firms. This benefit outweighs the higher taxation. The policy intervention reduces the welfare gap with respect to the first best allocation without taxation and with frictionless capital markets by 16.8%.

The last panel of Figure 9 shows the return on equity: $\frac{1}{e'} \int_{\tilde{\varepsilon}}^{\infty} q' \varphi(\varepsilon) d\varepsilon$. With decreasing returns in production, firm profits are positive. While creditors break even, shareholders earn an excess return over the riskless rate. The return on equity falls on impact in $t_p = 100$. As the long-term debt share is falling, transaction costs on the bond market increase. This effect is strongest right after t_p when leverage is still high. Shareholders pay these transaction costs through an increase of book equity e' . This reduces the return on equity. But this initial drop is followed by a fast rebound. As leverage falls, the initial increase of transaction costs on the bond market and the associated increase in e' are partially reversed. And with a lower stock of outstanding long-term debt, the firm internalizes a larger fraction of the total benefit of reduced debt dilution in the form of lower bond spreads.

Policy 2: Measuring the Cost of Debt Dilution

We have seen that an outright ban of long-term debt improves the allocation of capital and increases welfare in our model economy. But given that long-term debt saves transaction costs on the bond market, there might be better ways to address firms' time-inconsistency problem. We conduct a second policy experiment to measure the maximum welfare gain which could be achieved by a complete neutralization of debt dilution. We compare the results from our benchmark economy with long-term debt and debt dilution to an alternative economy in which long-term debt is used but debt dilution is absent. In this alternative economy, firms are obliged to compensate creditors for all fluctuations in the value of outstanding long-term bonds which are caused by future borrowing. As we have seen in Section 4, this forces firms to internalize the value of outstanding long-term debt.²⁰

In order to know the correct compensation payment, the first step is to calculate the value of outstanding long-term debt in the absence of additional borrowing by the firm. Consider the following problem:

$$\hat{V}(b, z) = \max_{k, \bar{\varepsilon}} \quad -k + \frac{1}{1+r} \left[\int_{\bar{\varepsilon}}^{\infty} \left[q' + \mathbb{E}_{z'|z} \hat{V}((1-\gamma)b, z') \right] \varphi(\varepsilon) d\varepsilon \right. \\ \left. + \Phi(\bar{\varepsilon}) \mathbb{E}_{z'|z} \hat{V}(0, z') \right] \quad (45)$$

$$\text{subject to: } q' = k - \gamma b + (1-\tau)[zk^\alpha - \delta k + \varepsilon k - cb]$$

$$\bar{\varepsilon} : q' + \mathbb{E}_{z'|z} \hat{V}((1-\gamma)b, z') = \mathbb{E}_{z'|z} \hat{V}(0, z').$$

This is the problem of a firm which inherits an amount b of long-term debt from the past, but which is not allowed to sell or buy any bonds, neither short-term bonds nor long-term bonds. Capital is entirely financed by equity: $k = e'$. As in the model economies above, creditors pay a break-even price \hat{p}^L for a long-term bond of this firm. This price \hat{p}^L is the reference price which we use to compute the correct compensation payment to the holders of outstanding long-term bonds.

After we have solved problem (45) and calculated the reference price \hat{p}^L , we can study the problem of a firm in our alternative economy in which firms must compensate creditors for all fluctuations in the value of outstanding long-term bonds which are caused by future borrowing. As in our benchmark economy, firms finance capital through equity, short-term debt, and long-term debt. But now long-term bonds feature a covenant which specifies a compensation payment from the firm to the holders of long-term debt. This compensation payment depends on the difference between the reference price of long-term bonds \hat{p}^L and the actual price \hat{p}^L . It is equal to: $(\hat{p}^L - \hat{p}^L)b$. The firm's problem

²⁰In models of sovereign default and long-term debt, similar compensation payments are considered by Chatterjee and Eyigungor (2012) and Hatchondo et al. (2016).

is given by

$$\hat{V}(b, z) = \max_{k, e', \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}} - e' + \frac{1}{1+r} \left[\int_{\bar{\varepsilon}}^{\infty} \left[q' + \mathbb{E}_{z'|z} \hat{V}((1-\gamma)\tilde{b}^L, z') \right] \varphi(\varepsilon) d\varepsilon + \Phi(\bar{\varepsilon}) \mathbb{E}_{z'|z} \hat{V}(0, z') \right] \quad (46)$$

$$\text{subject to: } q' = k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)[zk^\alpha - \delta k + \varepsilon k - c(\tilde{b}^S + \tilde{b}^L)]$$

$$\bar{\varepsilon} : q' + \mathbb{E}_{z'|z} \hat{V}((1-\gamma)\tilde{b}^L, z') = \mathbb{E}_{z'|z} \hat{V}(0, z')$$

$$k = e' + p^S \tilde{b}^S + \hat{p}^L (\tilde{b}^L - b) - (\hat{p}^L - \hat{p}^L) b - H(\tilde{b}^S, \tilde{b}^L, b).$$

The compensation payment $(\hat{p}^L - \hat{p}^L)b$ from the firm to the holders of outstanding long-term debt enters the firm's problem through the constraint which specifies k as a function of the choice variables e' , \tilde{b}^S , and \tilde{b}^L . For given values of k , \tilde{b}^S , and \tilde{b}^L , a positive compensation payment to creditors increases the amount of equity shareholders have to inject into the firm. The compensation payment also affects creditor's payoff. If a creditor buys a long-term bond today and the firm does not default, the market value of this long-term bond tomorrow is guaranteed to be equal to \hat{p}^L . Long-term creditors' break-even condition now reads as:

$$\hat{p}^L(s, \phi(s)) = \frac{1}{1+r} \left[[1 - \Phi(\bar{\varepsilon})] \left(\gamma + c + (1-\gamma) \mathbb{E}_{z'|z} \hat{p}^L(s', \phi(s')) \right) + \Phi(\bar{\varepsilon}) \frac{(1-\xi)\tilde{q}(s, \phi(s))}{\tilde{b}^S + \tilde{b}^L} \right]. \quad (47)$$

Figure 10 and 11 show firms' new equilibrium policies as functions of the exogenous state z and the stock of outstanding long-term debt b . We see a clear contrast to the equilibrium policies in our benchmark economy with debt dilution. As long as b is not too high, the default rate, bond spreads, leverage, and the share of long-term debt are all perfectly constant in z and in b . It follows that the counter-cyclical behavior of default, bond spreads, leverage, and debt maturity generated by our benchmark economy is exclusively driven by cyclical debt dilution.

We also see that the firm's maturity choice has become trivial. The transaction cost on the bond market gives an incentive to issue long-term debt instead of short-term debt. In the absence of debt dilution, there is no longer a downside to using long-term debt. Long-term debt completely dominates short-term debt in this economy without debt dilution.

For low values of b , the only choice variable which responds to changes in b is equity e' . The reason is simple. Consider capital k :

$$\begin{aligned} k &= e' + p^S \tilde{b}^S + \hat{p}^L (\tilde{b}^L - b) - (\hat{p}^L - \hat{p}^L) b - H(\tilde{b}^S, \tilde{b}^L, b) \\ &= e' + p^S \tilde{b}^S + \hat{p}^L \tilde{b}^L - \hat{p}^L b - H(\tilde{b}^S, \tilde{b}^L, b). \end{aligned} \quad (48)$$

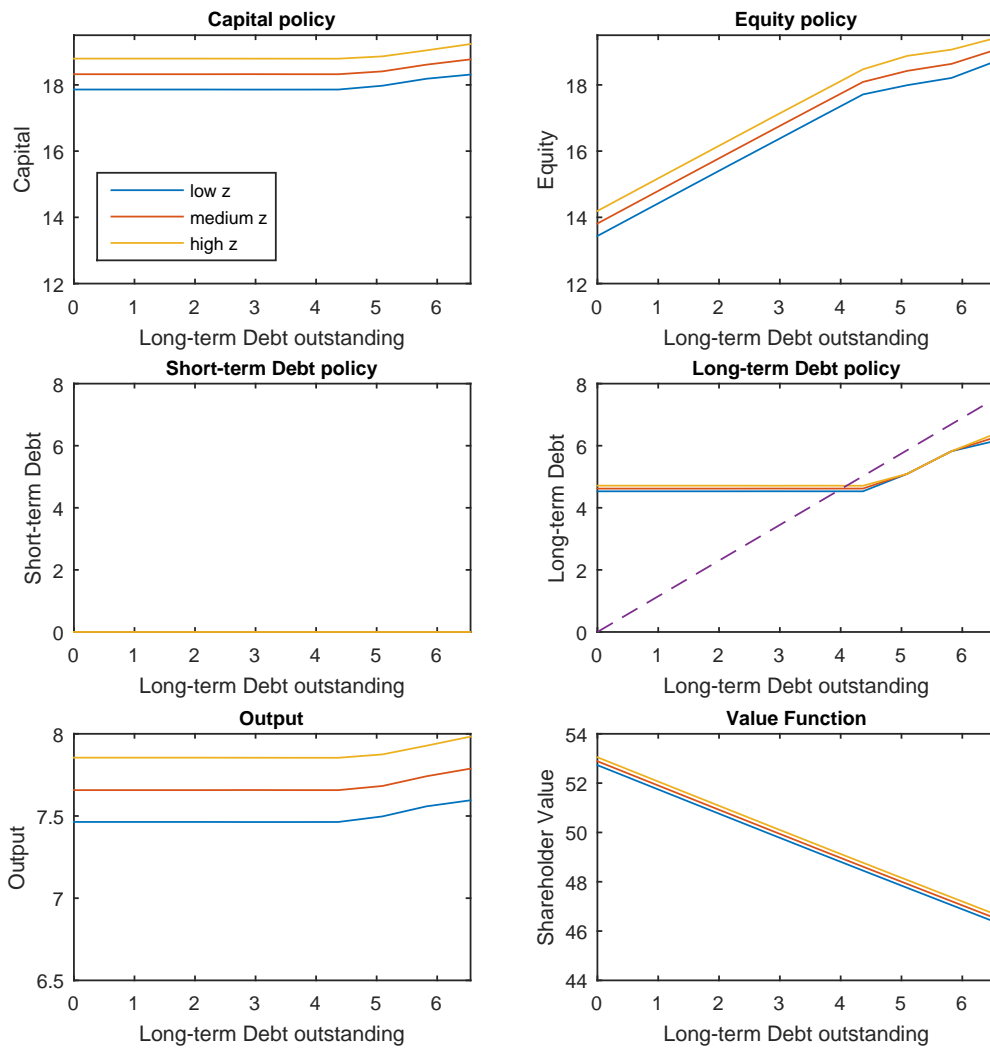


Figure 10: Debt Covenants - Policy Functions Part I.

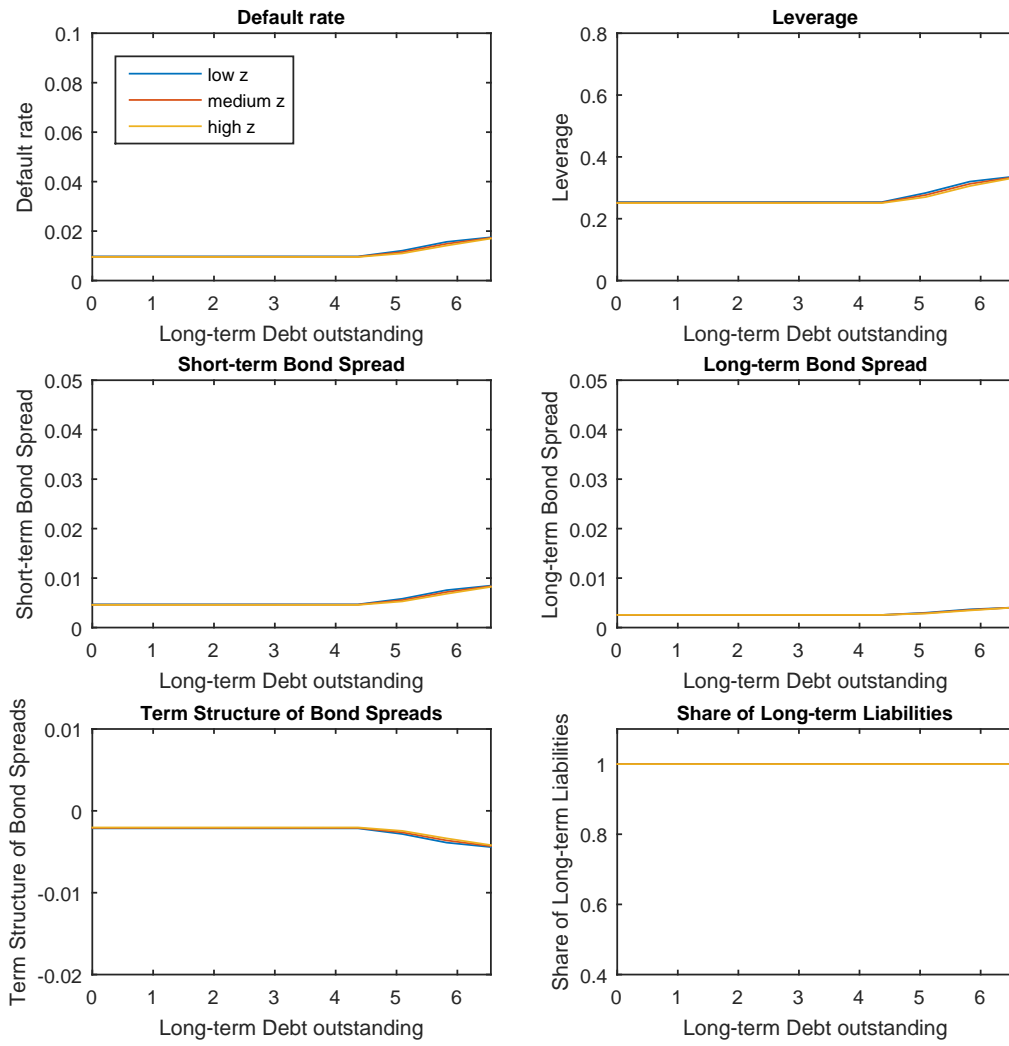


Figure 11: Debt Covenants - Policy Functions Part II.

As long as b is not ‘too high’, the firm’s choice of k , \tilde{b}^S , \tilde{b}^L , p^S , and \hat{p}^L is independent of b . The reference price \hat{p}^L is falling in b , but overall the reference value of outstanding debt $\hat{p}^L b$ increases in b . It follows that e' must rise. As the stock of outstanding debt b increases, less fresh capital can be raised on the bond market for a given desired level of leverage. Shareholders have to make up for the difference by injecting additional equity into the firm.

In the right panel of the middle row of Figure 10, we see that firms always choose to stay within the lower range of b where the default rate, bond spreads, leverage, and the share of long-term debt are constant in z and b . But why does the firm policy change for high values of b ? The firm maximizes the total return to capital. As b rises, more of the firm’s earnings are promised to the holders of outstanding long-term bonds and less is available to compensate agents who provide new capital: the shareholders and the buyers of newly issued debt. This makes the firm less profitable. As b rises beyond a certain point, the marginal unit of capital required to maintain a constant risk of default would earn a negative expected return. This marginal unit of capital is provided by shareholders as b is already above the desired level. At this point, the firm decides to inject less additional equity than needed to maintain a constant risk of default. The risk of default, credit spreads, and leverage begin to increase together with b .

As in the first policy experiment, we consider the unexpected introduction of the debt covenant in period $t_p = 100$. We assume that all outstanding long-term bonds without covenants in t_p are swapped with new long-term bonds which feature the covenant. Figure 12 shows what happens in our model economy after the policy intervention at time $t_p = 100$.

Compared to the ban of long-term debt considered in the first policy experiment, the response of the default rate, leverage, and bond spreads is more immediate. In the first policy experiment, debt dilution was gradually phased out as long-term debt was slowly repaid over time. Now we study an immediate and complete neutralization of debt dilution. Firms’ equilibrium policy changes on impact. The new default rate is about 1% compared to almost 3% before the policy intervention. Leverage falls from 43% to 25.2%, credit spreads from 1.5% to 0.24%. It follows that debt dilution alone accounts for 84% of the credit spread. Although leverage and the default rate are higher than after the ban of long-term debt studied above, the spread is lower. This is because the liquidation value in case of default is higher now (because $\bar{\varepsilon}$ is higher). While the share of long-term debt jumps up to 1 after the introduction of the debt covenant, the absolute amount of long-term debt falls.

As in the first policy experiment, output, capital, and welfare all increase in response to the introduction of the debt covenant. Compared to the ban of long-term debt, leverage falls by less and credit spreads fall by more. Together, this implies that output, capital, and welfare all increase more strongly compared to the ban of long-term debt. Compensation payments are a more efficient way to tackle firms’ time-inconsistency problem as they allow long-term debt to play its socially useful role in reducing transaction costs on the bond market. The policy intervention reduces 25% of the total welfare gap with respect to the first best allocation without taxation and with frictionless capital markets.

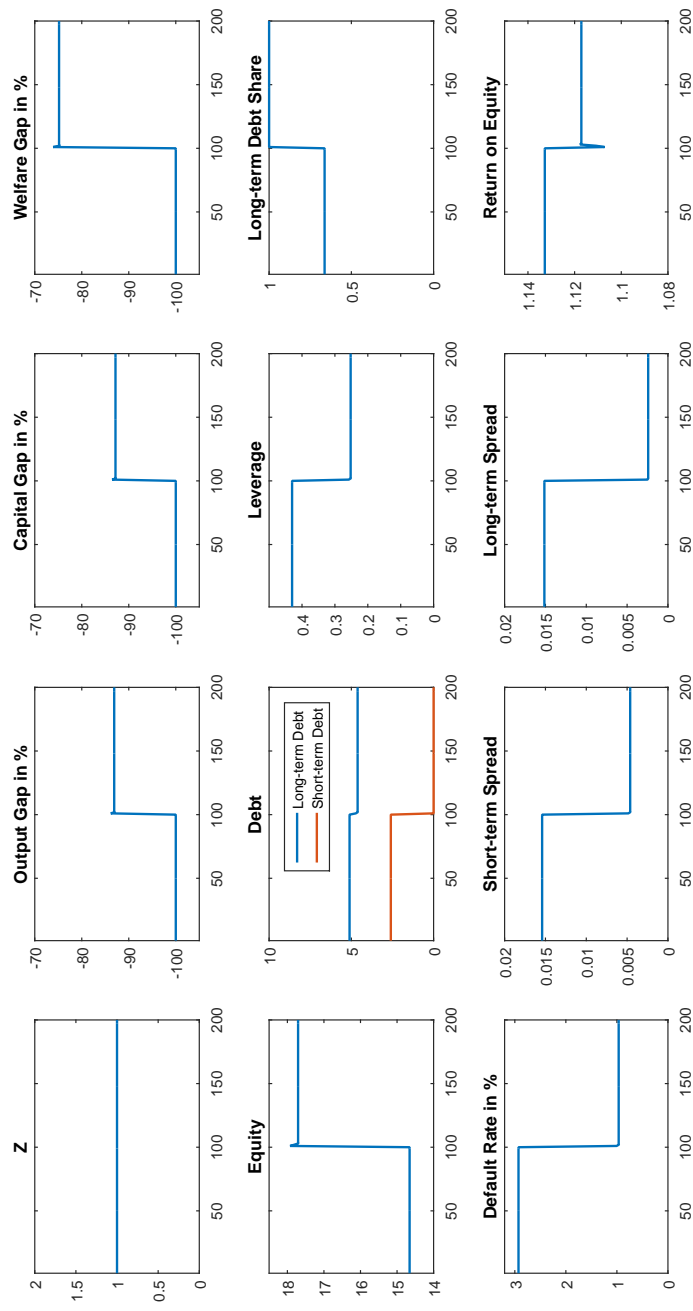


Figure 12: Compensation Payments.

The last panel of Figure 12 shows the return on equity: $\frac{1}{e'} \int_{\bar{\varepsilon}}^{\infty} q' \varphi(\varepsilon) d\varepsilon$. The return on equity falls on impact in $t_p = 100$. The sudden and unexpected introduction of the debt covenant imposes compensation payments on shareholders. For the holders of outstanding long-term debt this is a pure windfall profit as they charged a high credit spread when they bought long-term bonds in the past. The return on equity recovers quickly but never returns to its initial level before the policy intervention. The reason is that the total amount of equity e' increases substantially. Total dividend payments increase in response to the policy intervention.

A large part of the welfare gap with respect to the first best allocation is due to the corporate tax τ . Our parametrization uses as baseline value $\tau = 29.6\%$. In order to measure the cost of debt dilution, we calculate the reduction in τ which is just enough to generate the same welfare gain in our benchmark economy as the complete elimination of debt dilution. We find that the welfare gain from eliminating debt dilution corresponds to a tax cut of 2.5 percentage points.

Debt Covenants in Practice

We have seen that debt covenants can be a powerful instrument to contain debt dilution. They benefit both shareholders and the economy as a whole. While many different kinds of debt covenants are commonly included in publicly traded bonds, compensation payments like the ones considered in the policy experiment above are not used in practice.²¹ Debt covenants often prohibit the firm from issuing additional debt of higher seniority. However, covenants which restrict firms to issue additional debt of identical seniority (e.g. through leverage limits or minimum interest coverage ratios) are far less common.

Nash, Netter, and Poulsen (2003) find that 15.66% of 364 investment grade bond issues in 1989 and in 1996 feature restrictions on additional debt of identical seniority.²² In a sample of 100 bond issues 1999-2000, Begley and Freedman (2004) report that 9% contain additional borrowing restrictions.²³ Billett et al. (2007) calculate that 17.5% of 15,504 investment grade bond issued between 1960 and 2003 had a covenant which restricted future borrowing of identical seniority.²⁴ Reisel (2014) finds in a sample of 4,267 bond issues from 1989 - 2006 that 5.9% of investment grade bonds feature covenants which restrict additional borrowing.²⁵

In summary, the empirical literature on debt covenants finds that less than twenty percent of investment grade bonds issued by U.S. companies include covenants which restrict borrowing of identical seniority.

²¹Typical debt covenants are cross-default provisions which trigger default as soon as a firm defaults on another liability. Also, covenants commonly restrict the sale of certain firm assets or specify the status of debt in case of a merger. For an overview of the usage of different kinds of debt covenants, see for instance the descriptive statistics in Table III of Billett, King, and Mauer (2007).

²²Of the 496 bonds considered in their Compustat sample, 120 feature additional debt restrictions (Table 3, p. 218). Of those, 57 bonds are investment grade (Table 4, p.220). It follows that out of a total of 364 investment grade bonds (Table 2, p.216), 15.66% (or 57) feature additional debt restrictions.

²³See their Table 2, p. 24.

²⁴See their Table III, p. 707.

²⁵See her Table 4, p. 259.

6. Conclusion

This paper introduces a maturity choice to the standard model of firm financing and investment. We find that debt dilution plays an important role in the trade-off between short-term debt and long-term debt. The intensity of debt dilution varies over the business cycle. During recessions, firms disregard a larger fraction of the total potential costs of default. This mechanism generates the observed counter-cyclical behavior of default, bond spreads, leverage, and debt maturity. It also generates the pro-cyclical term structure of corporate bond spreads.

Debt dilution renders the equilibrium outcome constrained-inefficient. Credit spreads are too high and investment is too low. Given that long-term debt induces high default rates, firms would like to commit to a policy which uses a smaller amount of long-term debt in the future. But firms' time-inconsistency problem also affects their maturity choice. As the stock of outstanding long-term debt increases, firms internalize a smaller fraction of the entire value of debt. They disregard a larger fraction of the total future cost of debt dilution and choose a higher share of long-term debt. We show in our first policy experiment that a ban of long-term debt improves welfare in our model economy by reducing credit spreads and increasing investment.

In a second policy experiment, we compare the equilibrium of our benchmark economy to an alternative economy in which long-term bond contracts feature a debt covenant which eliminates debt dilution. We find that debt dilution alone accounts for 84% of the credit spread and 25% of the welfare gap with respect to the first best allocation. The welfare gain from eliminating debt dilution corresponds to a cut in the corporate income tax of 2.5 percentage points.

A number of unanswered questions remain. Most importantly, several key results of this paper could change once one considers a model economy with more heterogeneity in the firm distribution. In our model, most firms choose identical policies. In the data, leverage and the distance to default vary considerably across firms. Cooley and Quadrini (2001), Hennessy and Whited (2005), and Covas and Den Haan (2012) show how to replicate a number of these cross-sectional patterns using models with costly equity issuance.

Given our parametrization, cyclical debt dilution generates only a modest amount of amplification. We know that the amount of amplification in our model increases in the equilibrium probability of default. Since most firms in our model economy are identical and since the average rate of default in the data is low, it is not surprising that our model generates little amplification. This result could change in a model where firms are more different from the 'average firm'. In particular, cyclical debt dilution is likely to strongly affect firms with high leverage and low distance to default.

Also, firms' maturity choice might become richer in such a model. Long-term debt is a close substitute for equity in the sense that it generates a large inflow of cash today in return of small repayments which are extended over several time periods in the future. With costly equity issuance, long-term debt becomes more attractive because it is a better substitute for equity issuance than short-term debt. An outright ban of long-term debt might be more costly in a model with costly equity issuance.

In our model, a firm's leverage ratio only changes because of debt dilution. There might be additional reasons why a firm chooses to adjust leverage. A model economy which takes these additional reasons into account could also provide the appropriate laboratory to study the optimal design of debt covenants. Leverage limits or interest coverage ratios are beneficial to the extent that they reduce debt dilution. But they might generate costs as well if they constrain firms in selecting a time-varying level of leverage.

A. Proofs and Derivations

Proof of Proposition 4.1

We divide the first order condition associated to $\bar{\varepsilon}$ by k :

$$[1 - \Phi(\bar{\varepsilon})](1 - \tau) \frac{\tau c}{1 + (1 - \tau)c} - \varphi(\bar{\varepsilon})(1 + c) \left(\frac{1 + (1 - \tau) \left[z \frac{f(k)}{k} - \delta + \bar{\varepsilon} \right]}{1 + (1 - \tau)c} - \frac{b}{k} \right) = 0.$$

In case $b = 0$, this is equivalent to:

$$[1 - \Phi(\bar{\varepsilon})](1 - \tau)\tau c - \varphi(\bar{\varepsilon})(1 + c) (1 + (1 - \tau) [-\delta + \bar{\varepsilon}]) = \varphi(\bar{\varepsilon})(1 + c)(1 - \tau)z \frac{f(k)}{k}.$$

The first order condition associated to k can be re-arranged as:

$$\begin{aligned} -1 + \frac{1 + c}{1 + r} [1 - \Phi(\bar{\varepsilon})] \frac{1 + (1 - \tau)[- \delta + \bar{\varepsilon}]}{1 + (1 - \tau)c} + \frac{1 - \tau}{1 + r} \int_{\bar{\varepsilon}}^{\infty} [\varepsilon - \bar{\varepsilon}] \varphi(\varepsilon) d\varepsilon \\ = -\frac{1 + c}{1 + r} [1 - \Phi(\bar{\varepsilon})] \frac{(1 - \tau)}{1 + (1 - \tau)c} z f'(k). \end{aligned}$$

Using the property of $f(k)$ that the marginal product is a constant fraction of its average product, we can combine these two first order conditions. We are left with a single equation with $\bar{\varepsilon}$ as the only endogenous variable. Neither aggregate productivity z nor k appear in this equation. It follows that the optimal choice of $\bar{\varepsilon}$ and the the default rate $\Phi(\bar{\varepsilon})$ do not depend on z if $b = 0$. This implies that the bond price p and the credit spread $r_b - r$ are constant as well.

What about leverage?

$$\frac{\tilde{b}}{k} = \frac{1 + (1 - \tau) \left[z \frac{f(k)}{k} - \delta + \bar{\varepsilon} \right]}{1 + (1 - \tau)c}.$$

We know from the first order condition associated to $\bar{\varepsilon}$ that $z \frac{f(k)}{k}$ must be constant in z if $\bar{\varepsilon}$ does not move and $b = 0$. It follows that leverage $\frac{\tilde{b}}{k}$ is constant in z if $b = 0$.

Now we consider the case $b > 0$. The first order condition of k remains unchanged. With $b > 0$, there is an additional term added to the marginal benefit of $\bar{\varepsilon}$. This term is equal to:

$$\frac{b}{k} \varphi(\bar{\varepsilon}) (1 + c).$$

This term is positive and increasing in b . By adding this term to the firm's first order condition, the solution for $\bar{\varepsilon}$ is increased. The optimal choice of $\bar{\varepsilon}$ is raised by less if k is higher. Since the optimal choice of k is increasing in z , it follows that a high value of z implies a low value of $\bar{\varepsilon}$ if $b > 0$. It follows that $\bar{\varepsilon}$ and the default rate $\Phi(\bar{\varepsilon})$ are falling in z if $b > 0$. This implies that the bond price p rises and the credit spread $r_b - r$ falls

in z .

Proof of Proposition 4.2

We see immediately that b does not show up in the firm's problem. The first order condition for k is the same as in the case without compensation payments. The first order condition for $\bar{\varepsilon}$ is:

$$\bar{\varepsilon} : [1 - \Phi(\bar{\varepsilon})] \left[(1 + c) \frac{\partial \tilde{b}}{\partial \bar{\varepsilon}} - (1 - \tau)k \right] - \varphi(\bar{\varepsilon})(1 + c)\tilde{b} = 0.$$

It follows that a firm which needs to compensate creditors for changes in the value of previously issued bonds will take exactly the same decisions as a firm which does not have any long-term debt outstanding ($b = 0$).

B. Additional Quantitative Results

B.1. The Firm's Objective Function

Figure 13 shows the firm objective as a function of the firm's three choice variables: capital k , the threshold value $\bar{\varepsilon}$, and the long-term debt share m . The firm's state $(b, z) = (2.51, 1)$, tomorrow's price of long-term debt $p^L(s', \phi(s'))$, and tomorrow's firm policy $\phi(s')$ are taken as given. The firm's objective around the global optimum is strictly concave in all three choice variables. The three panels show the firm's objective as a function of k and $\bar{\varepsilon}$ (top), of k and m (middle), and of $\bar{\varepsilon}$ and m (bottom). In all three panels, the omitted third choice variable (m , $\bar{\varepsilon}$, and k , respectively) is held constant at its optimal value. The pink dot indicates the firm's choice.

B.2. A model without Long-term Debt

In Figure 14, we show the equilibrium policy functions of a model in which firms are not allowed to issue long-term debt. In the absence of long-term debt, the amount of debt issued in the previous period is irrelevant for the firm's optimal policy. Equity, debt, and capital are pro-cyclical just as in our benchmark model with long-term debt. Leverage, the default rate, and the bond spread are a-cyclical. The firms' choices are highly similar to the equilibrium policies from our benchmark economy for $b = 0$. Figure 15 shows impulse response functions after a negative drop in z .

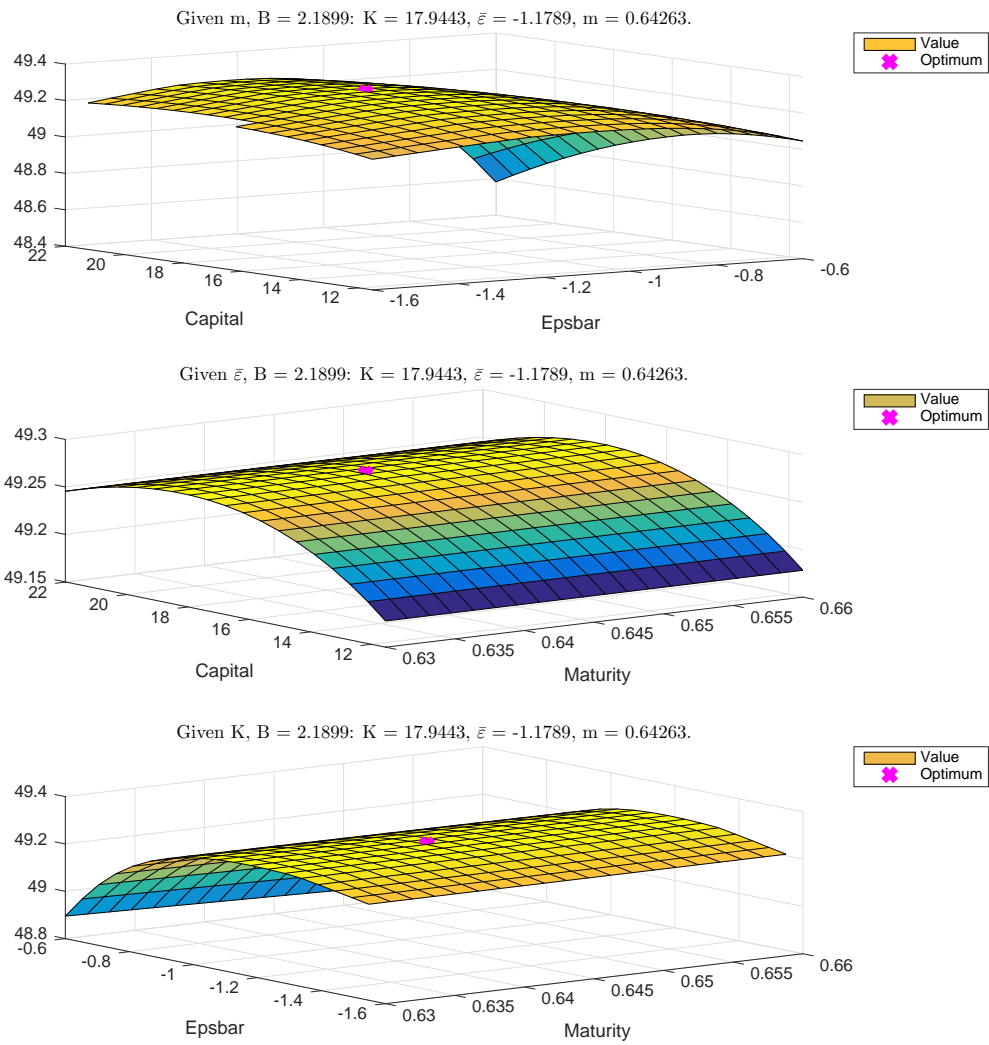


Figure 13: Firm Objective.

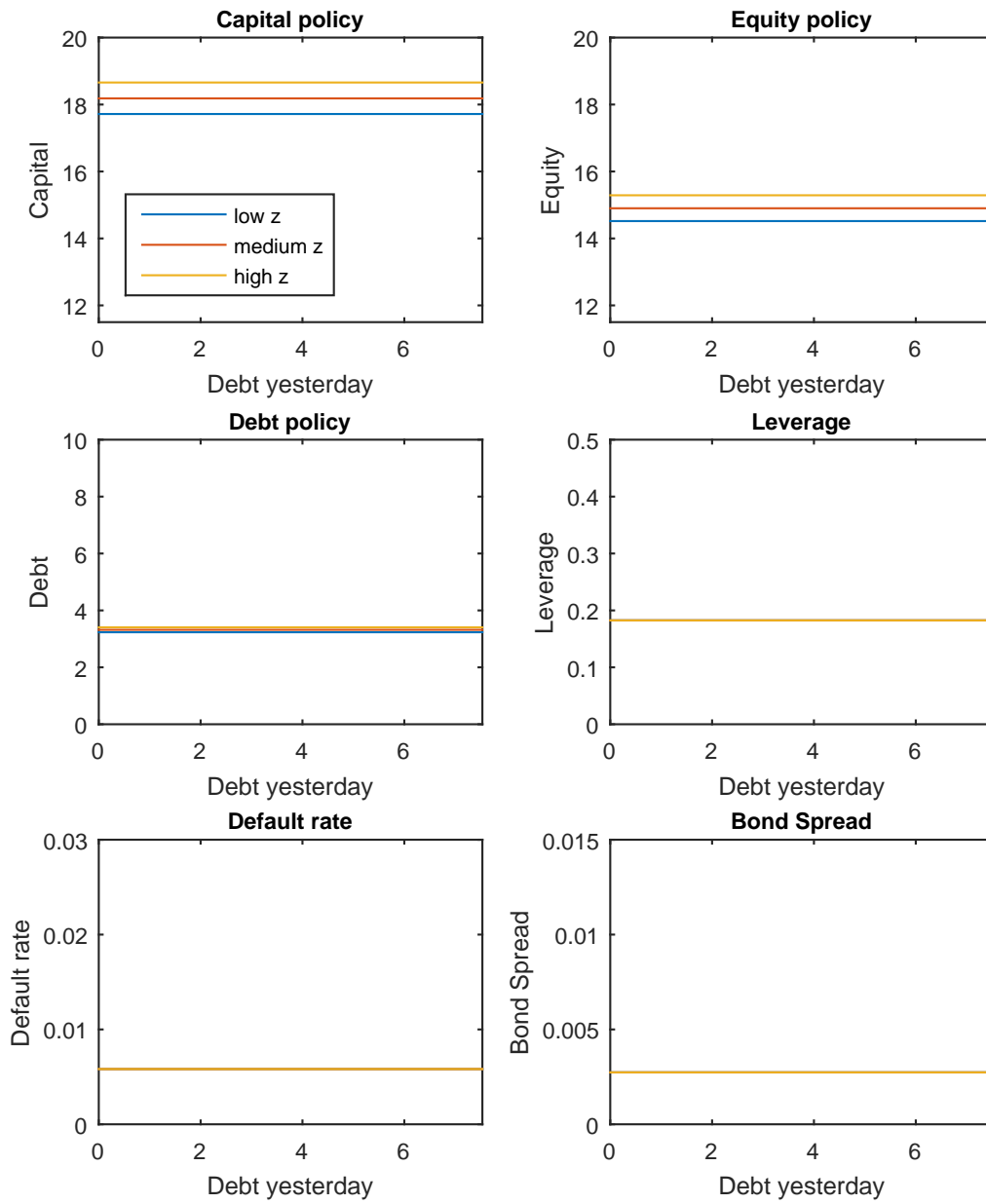


Figure 14: A model without Long-term Debt - Policy Functions

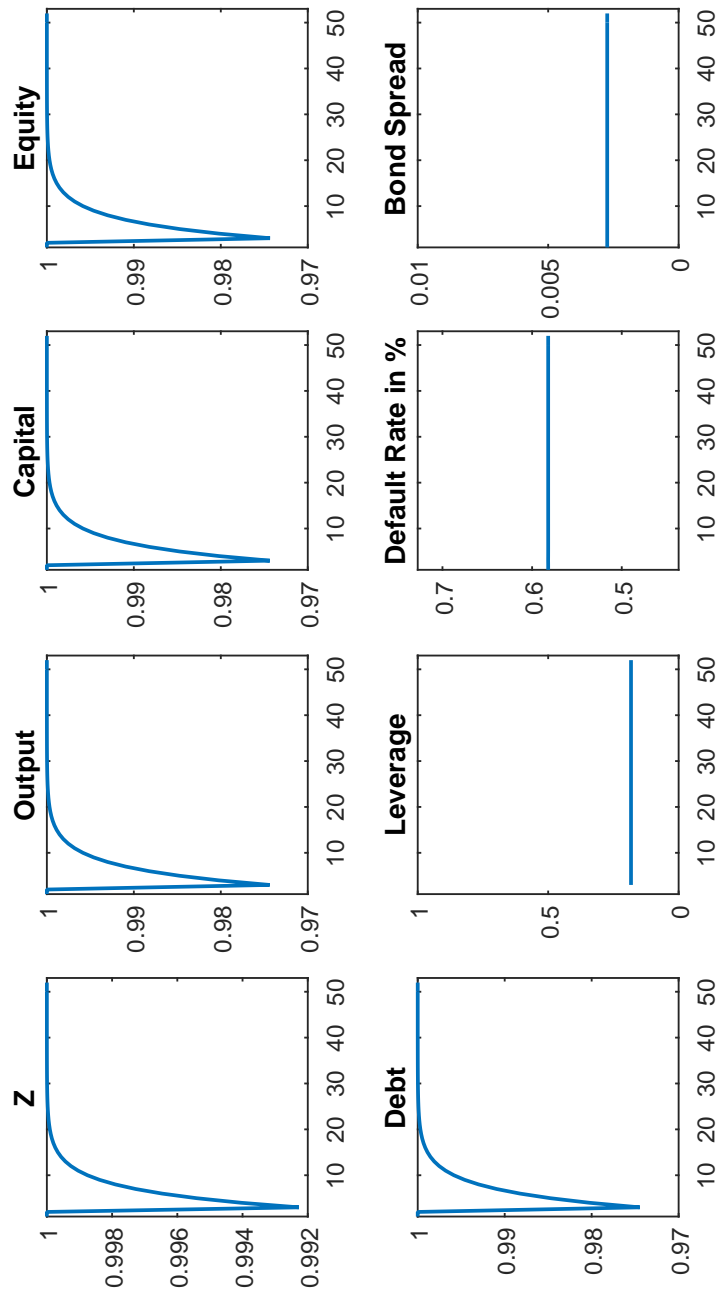


Figure 15: A model without Long-term Debt - Impulse Response Functions

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